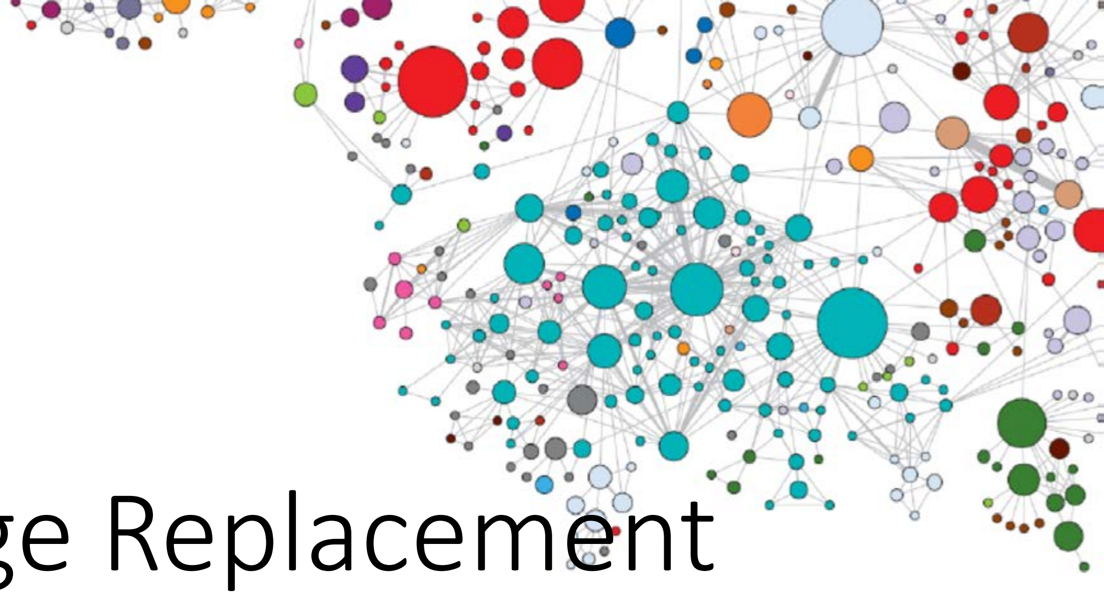

Synchronous Hyperedge Replacement Graph Grammars

Corey Pennycuff, Satyaki Sikdar, Catalina Vajiac,
David Chiang, and Tim Weninger

Department of Computer Science and Engineering

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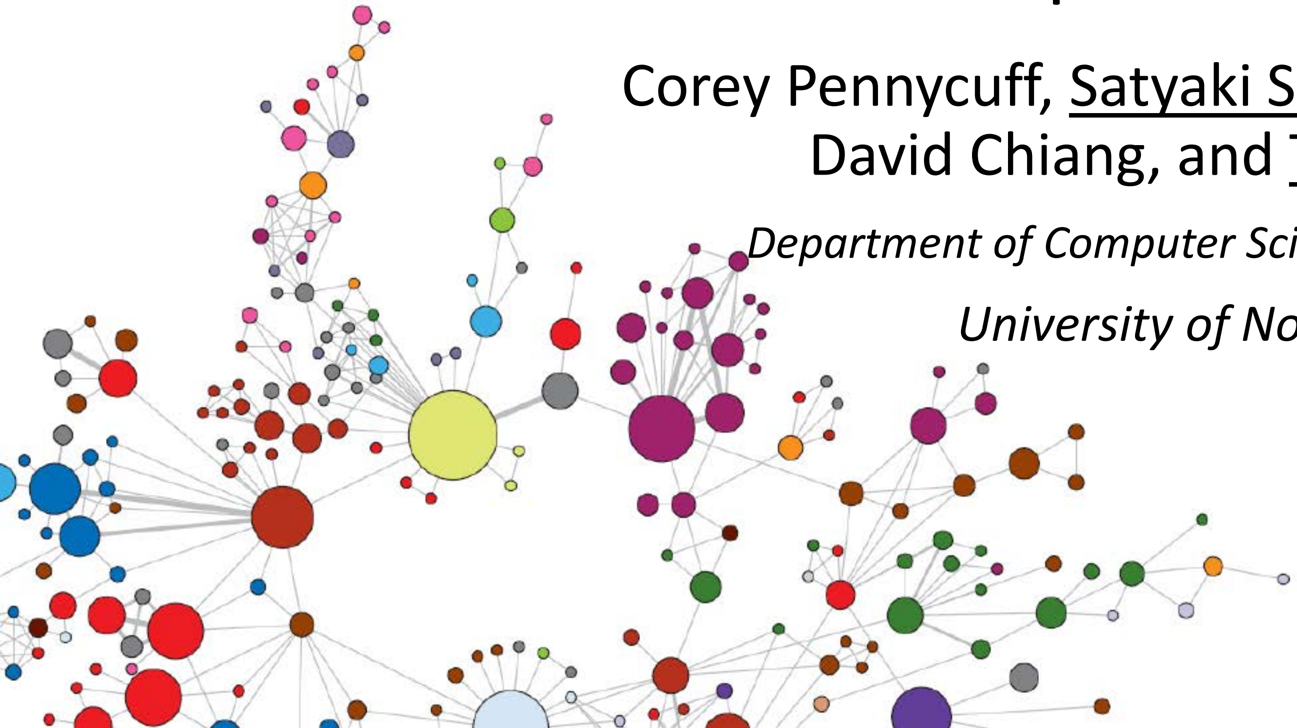


A brief reminder of Hyperedge Replacement Grammars

Context Free Grammars

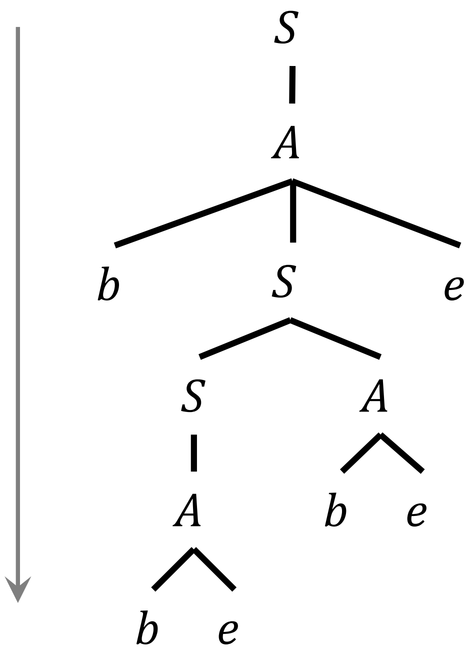
Production Rules

R1: $S \rightarrow SA$
R2: $S \rightarrow A$
R3: $A \rightarrow bSe$
R4: $A \rightarrow be$

Derivation of “$bbebee$”

	S
R2	A
R3	bSe
R1	$bSAe$
R2	$bAAe$
R4	$bbeAe$
R4	$bbebee$

Parse Tree



Context Free Grammars

Where did the Production Rules come from? Can we learn them?

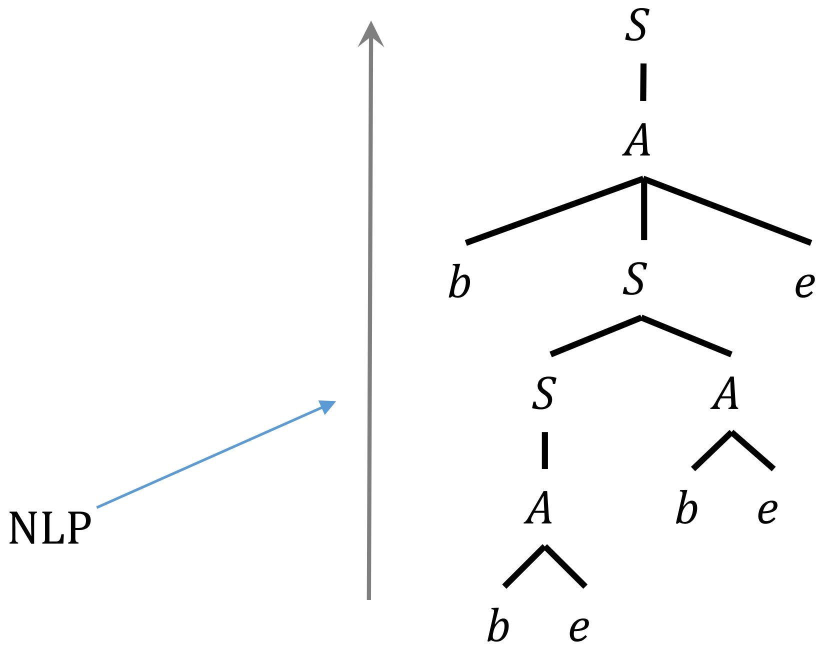
Production Rules

R1: $S \rightarrow SA$
R2: $S \rightarrow A$
R3: $A \rightarrow bSe$
R4: $A \rightarrow be$

Learning production rules from “*bbebee*”

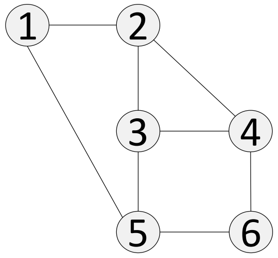
	S
R2	A
R3	bSe
R1	$bSAe$
R2	$bAAe$
R4	$bbeAe$
R4	$bbebee$

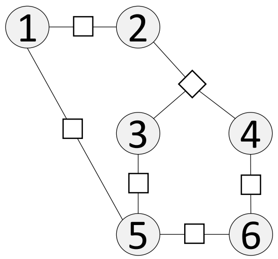
Parse Tree



Learning Hyperedge Replacement Grammars

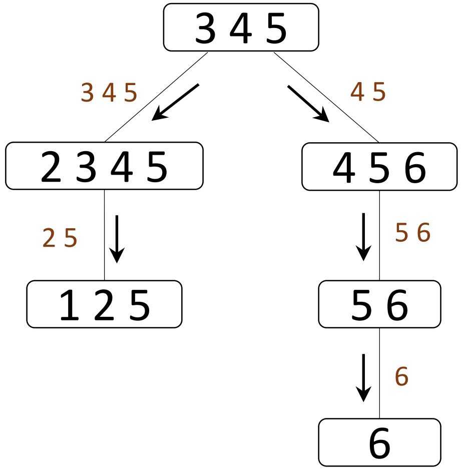
Graph





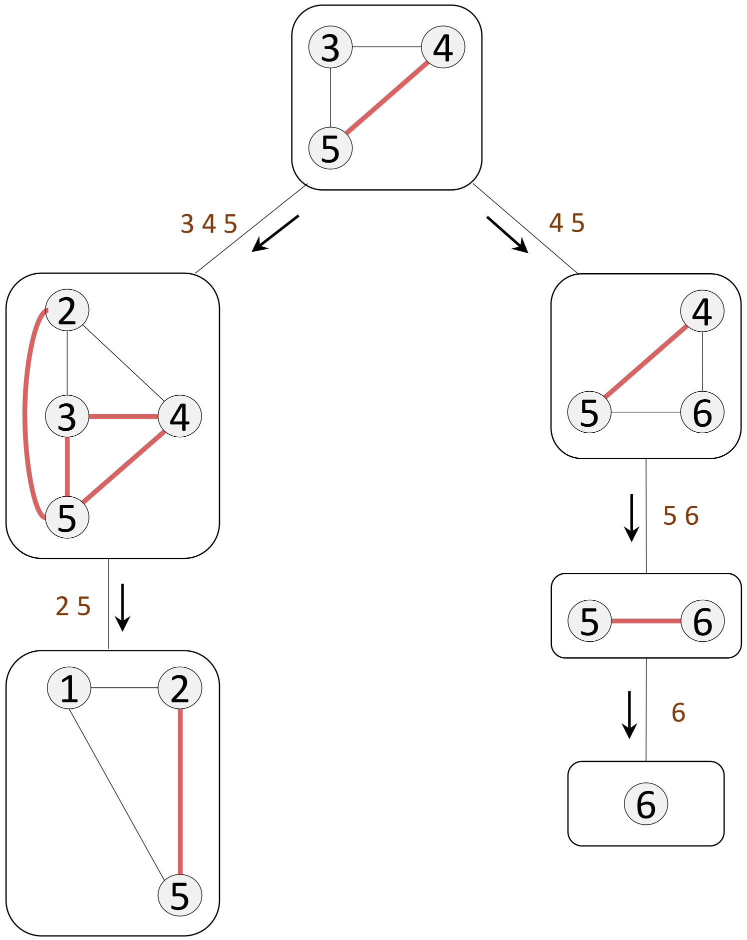
Tree Decomposition

used for many things:
Exact inference in probabilistic graphical models
Viterbi Algorithm runs on a tree decomposition

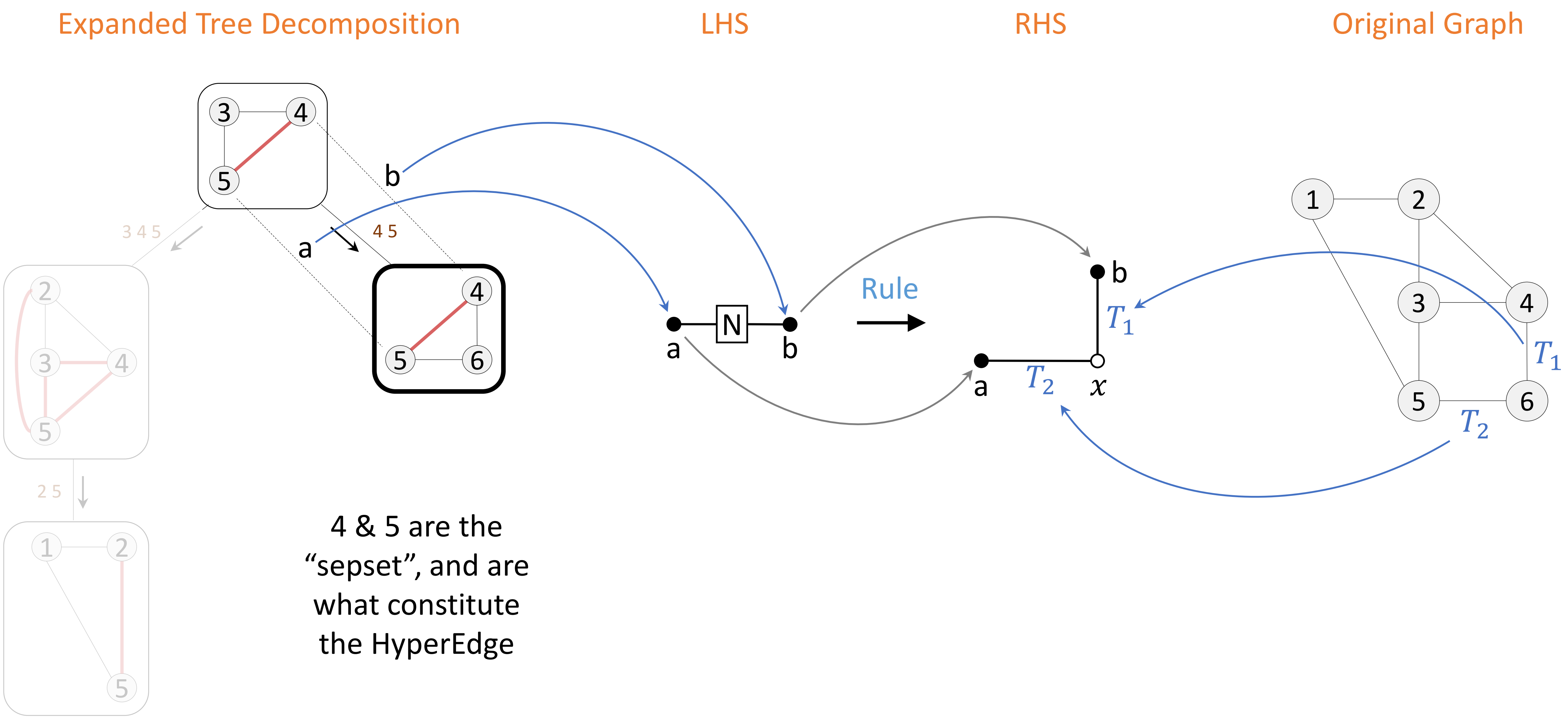


created via elimination ordering
Maximal Cardinality Search (MCS) Heuristic

Expanded Tree Decomposition

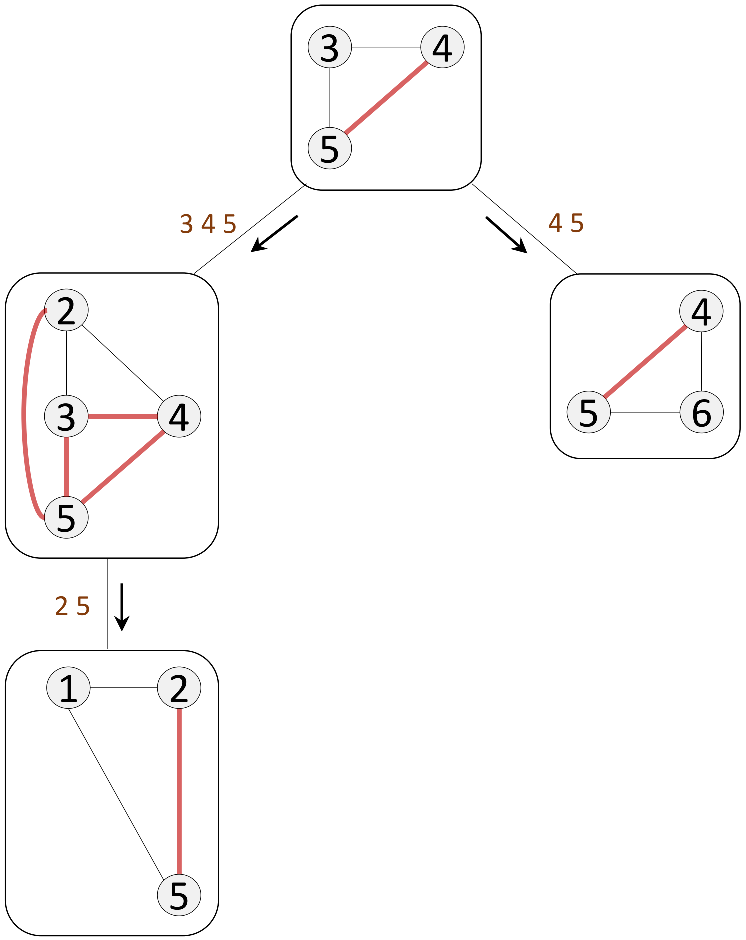


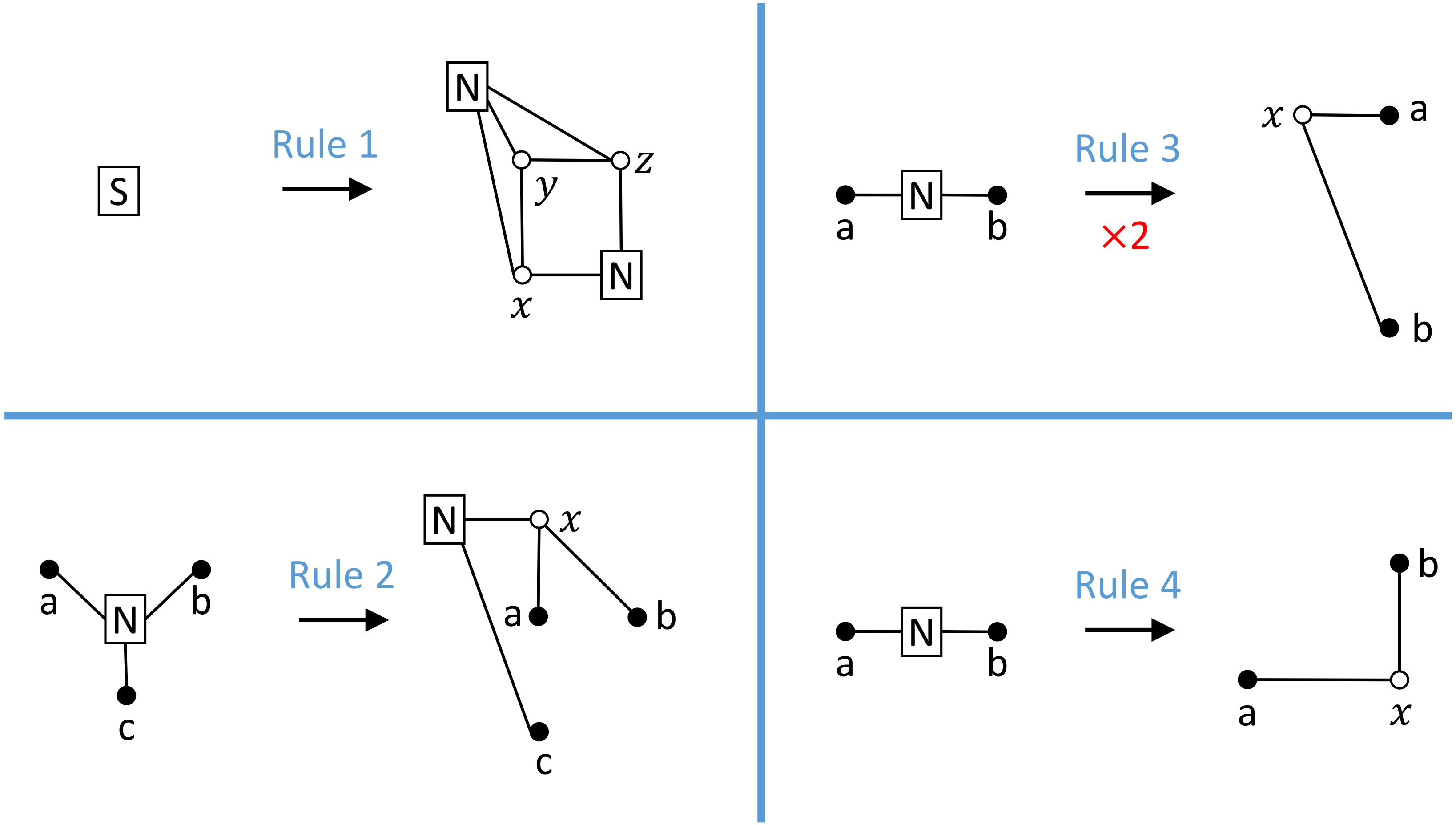
Learning Hyperedge Replacement Grammars



4 & 5 are the "sepset", and are what constitute the HyperEdge

The HRG

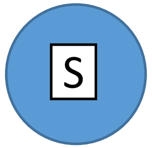

3
4
5
3 4 5
4 5
2
3
4
5
4
5
6
2 5
1
2
5


S
Rule 1
N
y
z
x
N
a
N
b
Rule 3
×2
x
a
b
a
N
b
c
Rule 2
N
x
a
b
c
a
N
b
Rule 4
b
a
x

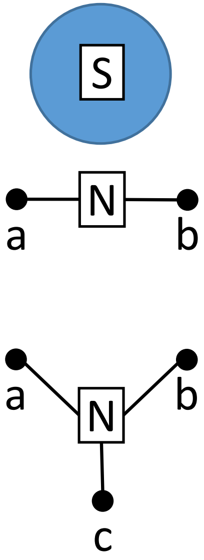
Growing a Graph

Always start with S

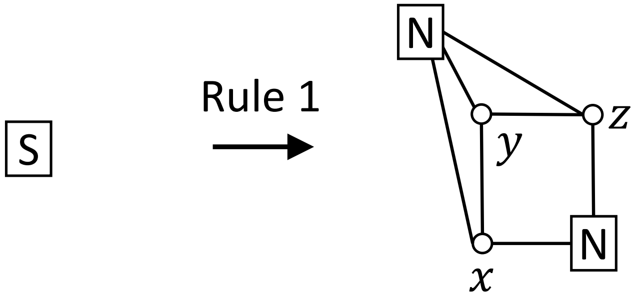
Current Graph



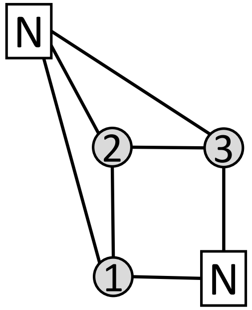
Match LHS



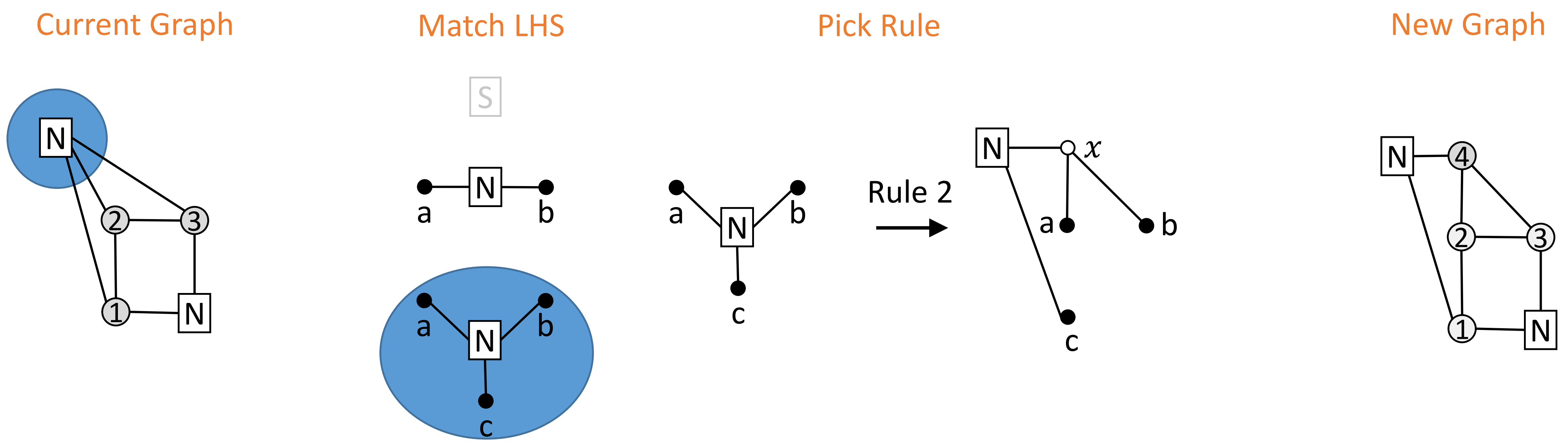
Pick Rule



New Graph

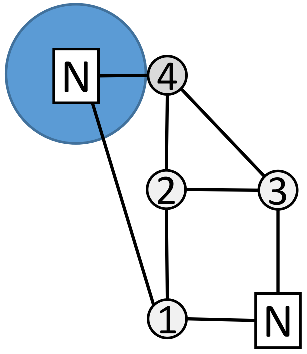


Growing a Graph

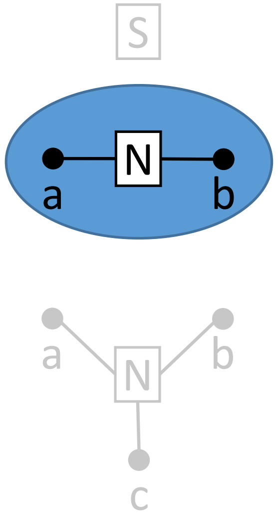


Growing a Graph

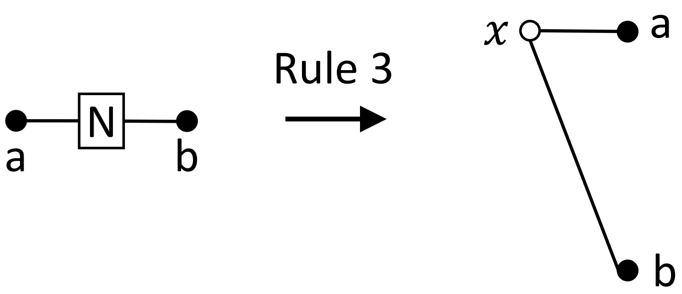
Current Graph



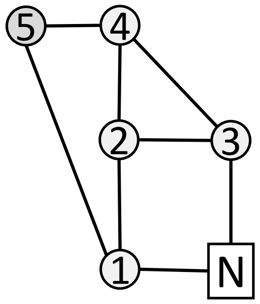
Match LHS



Pick Rule

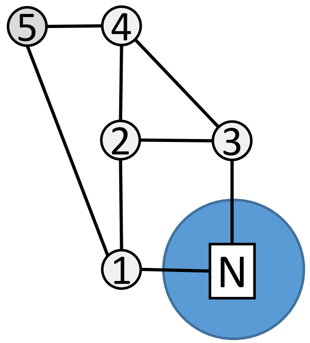


New Graph

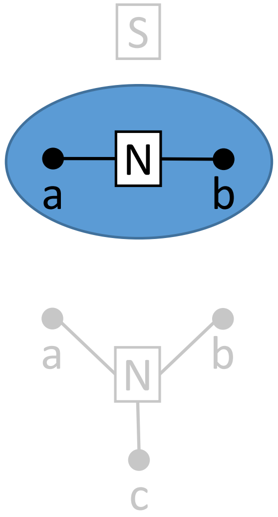


Growing a Graph

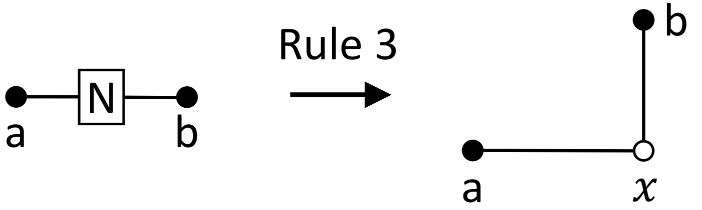
Current Graph



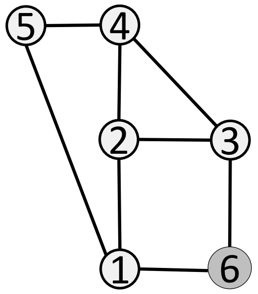
Match LHS



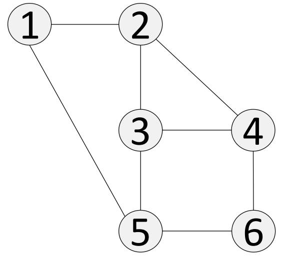
Pick Rule



New Graph



Original Graph



[PAMI 2018]

What’s Missing

HRG is not learned from the evolution of the actual graph

(Neither are most other graph generators)

- Tree Decomposition of the static, global graph is unnatural and clumsy
- Rules don’t *mean* anything

Synchronous HRGs

Synchronous CFGs

Given 2 equivalent sentences in different languages:

- English: `I open the box.`
- Japanese: `Watashi ha hako wo akemasu.`

- Synchronous grammars map the syntactic structure and vocabulary for each language, and pairs them into a single rule.
- A sentenced decomposed with one grammar can be reconstituted using the corresponding rules from the other language, and is thereby translated.

- How do we apply these to graphs?

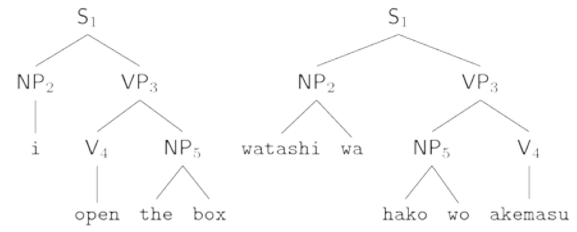
Production Rules

(LHS → $RHS_{English}$ | $RHS_{Japanese}$)

LHS	English	Japanese
S →	NP_1 VP_2	NP_1 VP_2
VP →	V_1 NP_2	NP_2 V_1
NP →	i	watashi ha
NP →	the box	hako wo
V →	open	akemasu

Applying Rules Synchronously

Rule	English	Japanese
	S_1	S_1
1	NP_2 VP_3	NP_2 VP_3
2	NP_2 V_4 NP_5	NP_2 NP_5 V_4
3	i V_4 NP_5	watashi ha NP_5 V_4
4	i V_4 the box	watashi ha hako wo V_4
5	i open the box	watashi ha hako wo akemasu



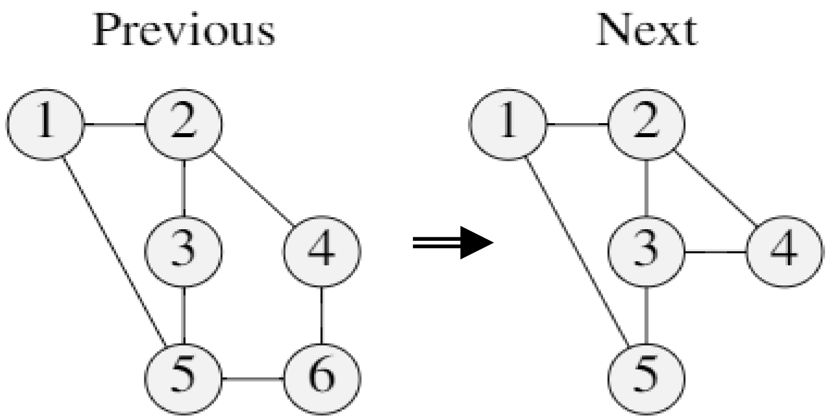
Example from Chiang, D.: An Introduction To Synchronous Grammars

Applying SCFGs to Graphs (Intuition)

- How can we "Translate" a graph?
  - We translate from one timestep to the next

- English : Japanese :: $H^{(t)} : H^{(t+1)}$

I open the box. ⟹ Watashi ha hako wo akemasu.

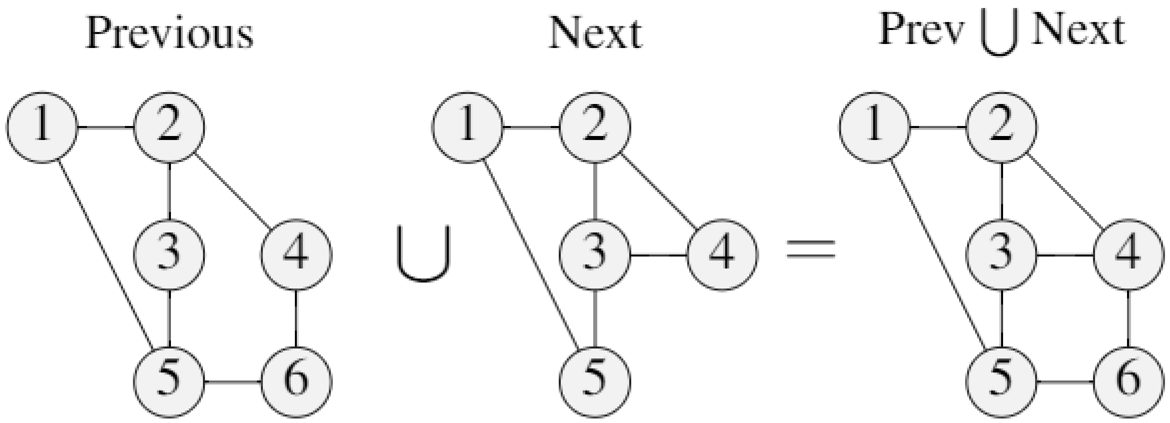


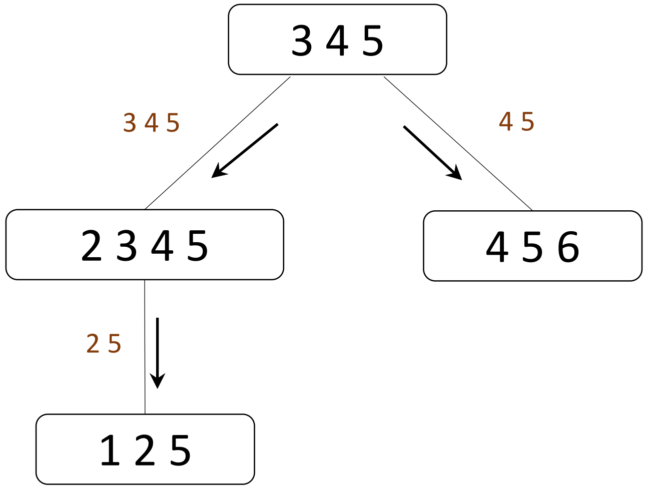
Tree Decomposition from the Union

How can two distinct, temporal snapshots of a graph be "equivalent"?

Work from a union of the two graphs

- Union of Graphs



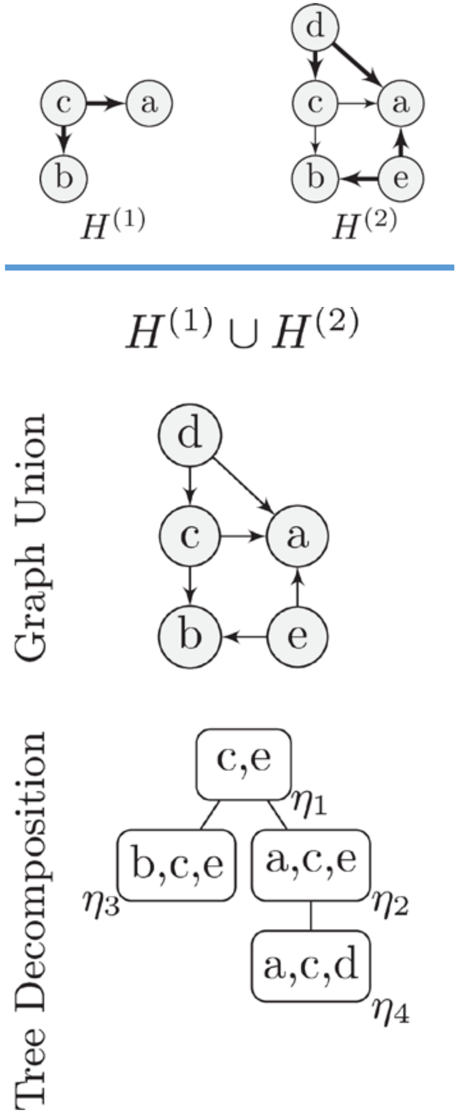


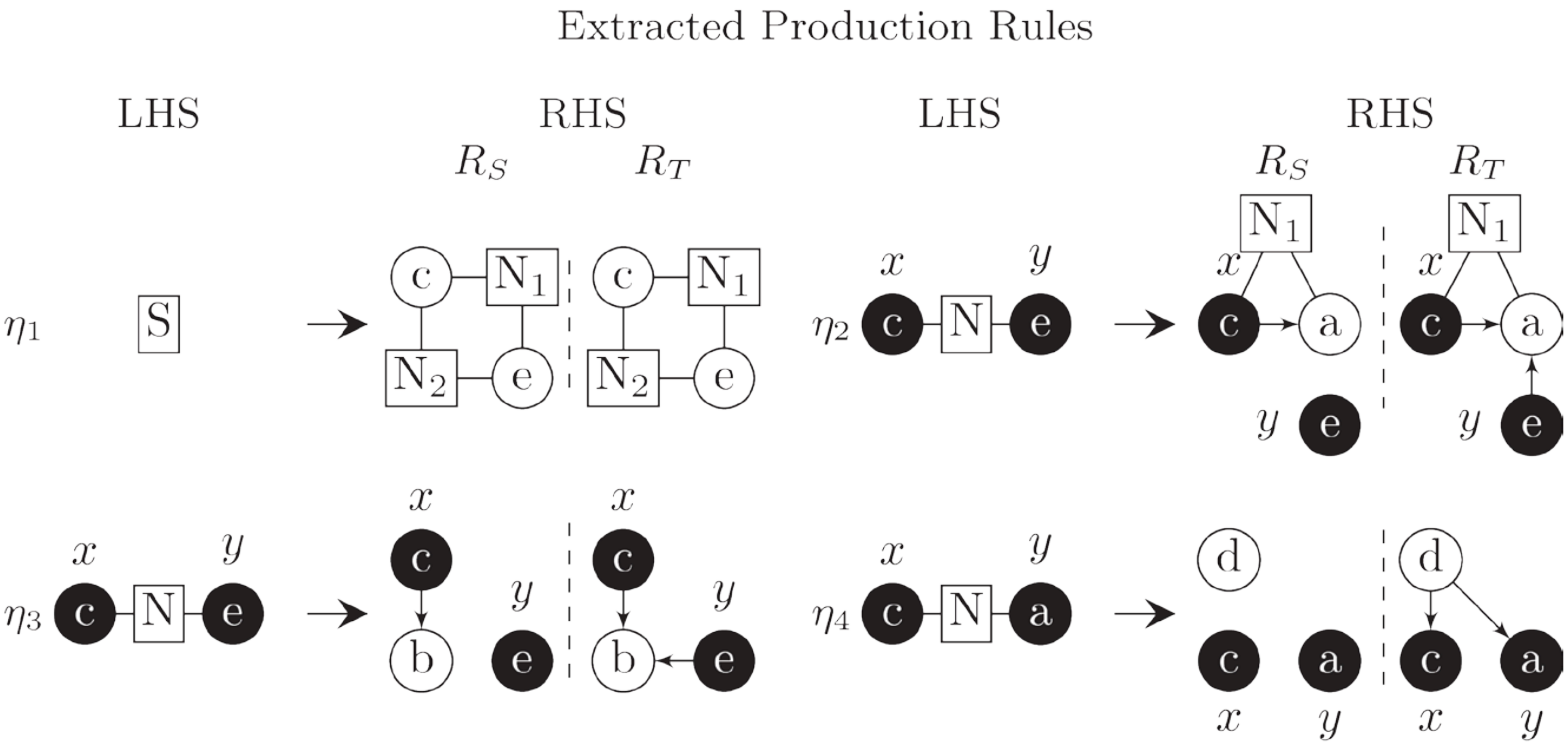
Generate synchronous rules from the context of the individual timestamps

Generated rules must have identical size and number of non-terminals in each paired rule.

Differences in a synchronous rule can model addition or removal of edges.

Extracting a Synchronous Hyperedge Replacement Grammar





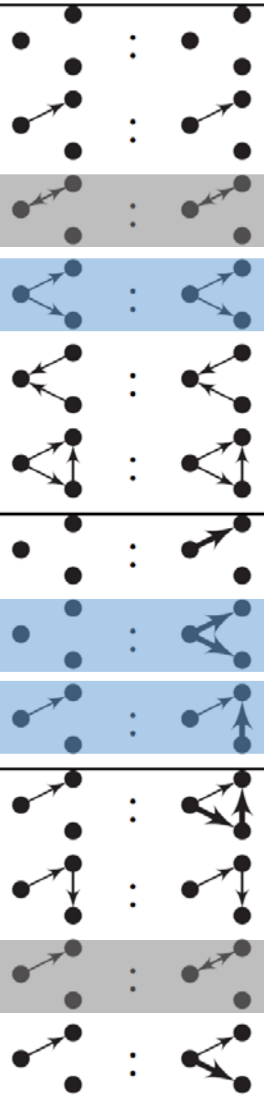
Is the SHRG *Meaningful*?

Generate 1000 of each Graph Type

- BA with $n = 10$ and $m = 2$, over $n - m$ timesteps
- ER: create n vertices & add edges between two vertices with probability p
  - ER_1: $n(n-1)$ timesteps, with $n(1-p)$ timesteps expected to contain no changes
  - ER_2: same as ER_1, but skips timesteps where no changes occur
  - ER_3: create 2 directed edges per timestep
    - $pn(n-1)$ edges over $pn(n-1)/2$ timesteps

Inspect rules to see if consistent with BA growth process

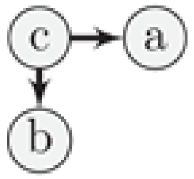
- Rule 4 – persistence of wedges
- Rule 8 – creation of wedges
- Rule 9 – Preferential attachment
- Rules 3 & 12 – Impossible for BA



		R_S		R_T	BA	ER_1	ER_2	ER_3
Static Rules	1		:		0.051	0.157	0.171	0.153
	2		:		0.133	0.545	0.542	0.412
	3		:		0.000	0.006	0.017	0.007
	4		:		0.260	0.047	0.034	0.029
	5		:		0.032	0.028	0.030	0.017
	6		:		0.025	0.009	0.000	0.002
BA Rules	7		:		0.243	0.164	0.166	0.325
	8		:		0.097	0.000	0.000	0.001
	9		:		0.155	0.012	0.004	0.014
Other Rules	10		:		0.001	0.000	0.000	0.002
	11		:		0.000	0.021	0.031	0.028
	12		:		0.000	0.006	0.006	0.008
	13		:		0.000	0.006	0.005	0.007

Can we Predict Future Graph Changes?

Current Graph

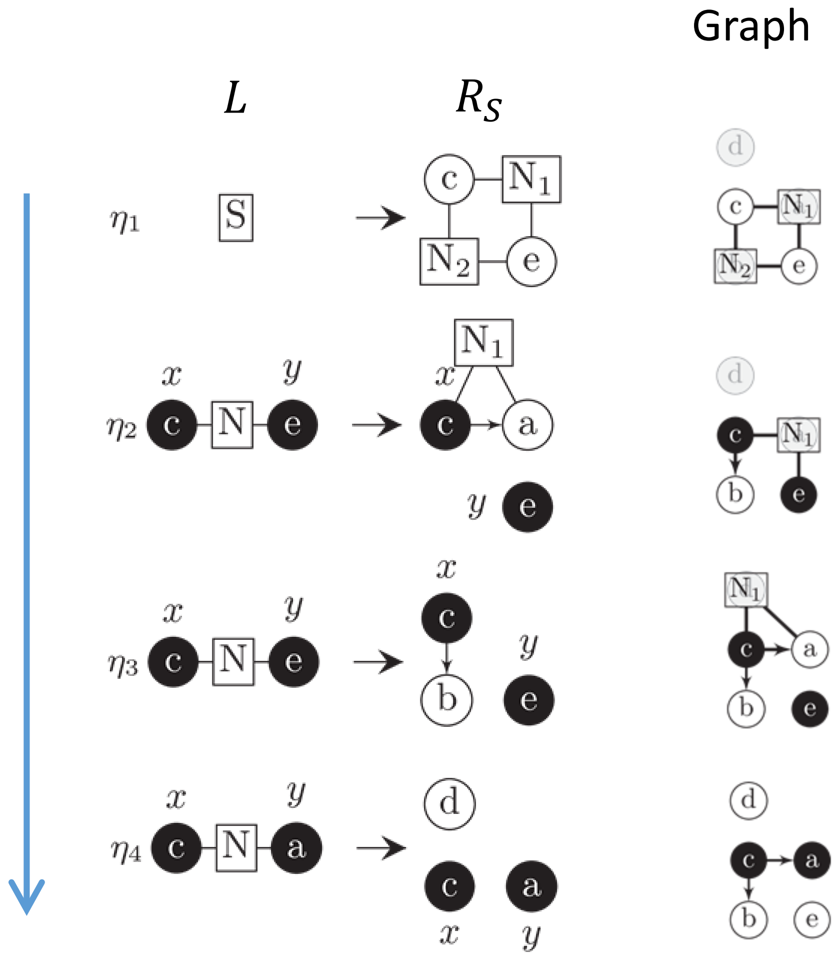


Parse Current Graph using R_S

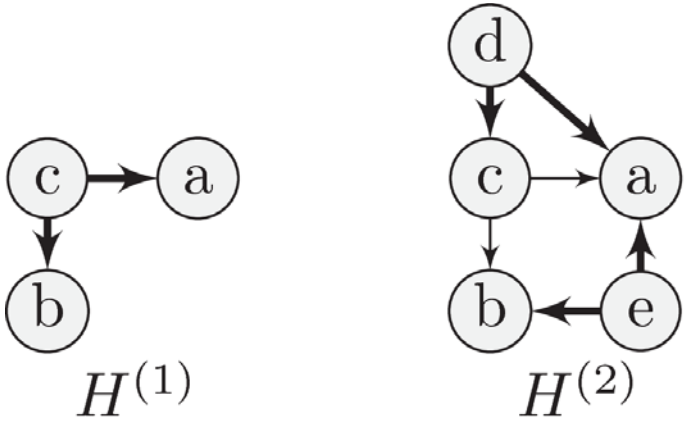
Algorithm adapted from hypergraph parsing algorithm (reverse-CYK algorithm) by Chiang et al (2013)

Algorithm produces a rule ordering (π) which can be used to generate the Source graph

$\pi =$ R1, R2, R3, R4



Generating $t + 1$

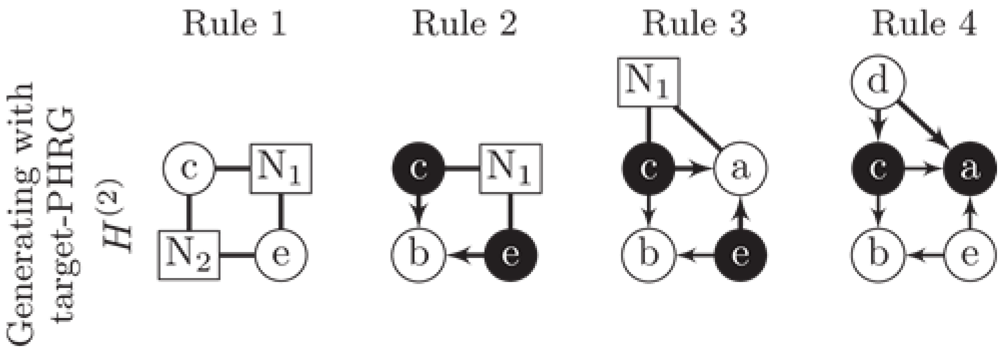


Generate $H^{(t+1)}$ using R_T

- Use π as the rule ordering, applying RHS rules R_T

Limitations

- Cannot predict unseen structures
- Multiple (valid & optimal) π
- Hypergraph parsing limits graph size (computationally)



Experiments

Methodology

Given a dynamic graph H with n timesteps, extract PSHRG grammar from $H^{(1)} \ldots H^{(n-1)}$.

- Extract π using Chiang algorithm and R_S on $H^{(n-1)}$
- Execute π ordering with R_T, creating H*
- Compare H* to $H^{(n)}$

Graphs are small

- $5 - 12$ nodes
- Limitation of parsing tools

Cramér–von Mises Statistic

- For comparing distributions

Repeat 50 times and plot the mean

Comparisons

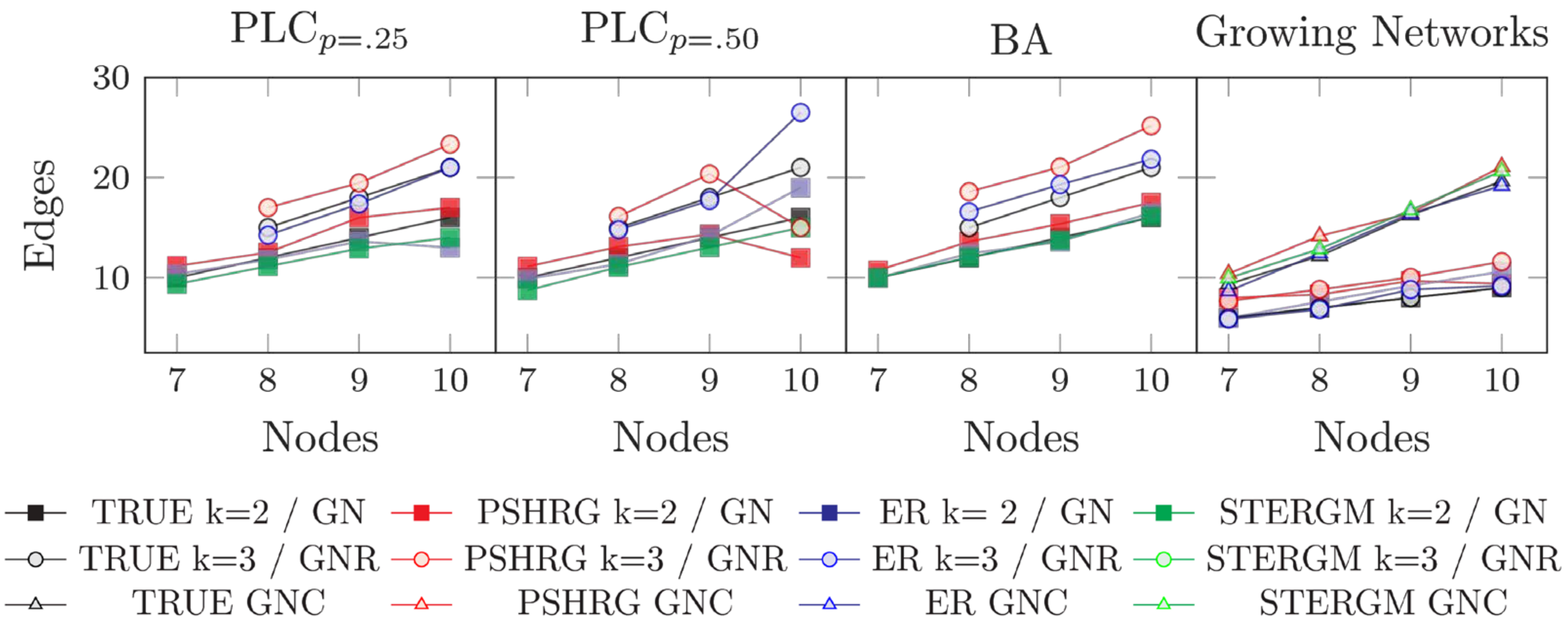
- In-degree
- Out-degree
- PageRank
- Graphlet Correlation Distance (GCD)

Other Graph Generators

- Barabási–Albert (BA) model
  - $k = 2$ and $k = 3$
- Powerlaw-Cluster graph (PLC)
  - $p = 0.25$ and $p = 0.5$
- “Growing networks”
  - GN, GNR, and GNC, $p = 0.5$
- Erdős–Rényi
- Separable Temporal Exponential Random Graph Model (STERGM)

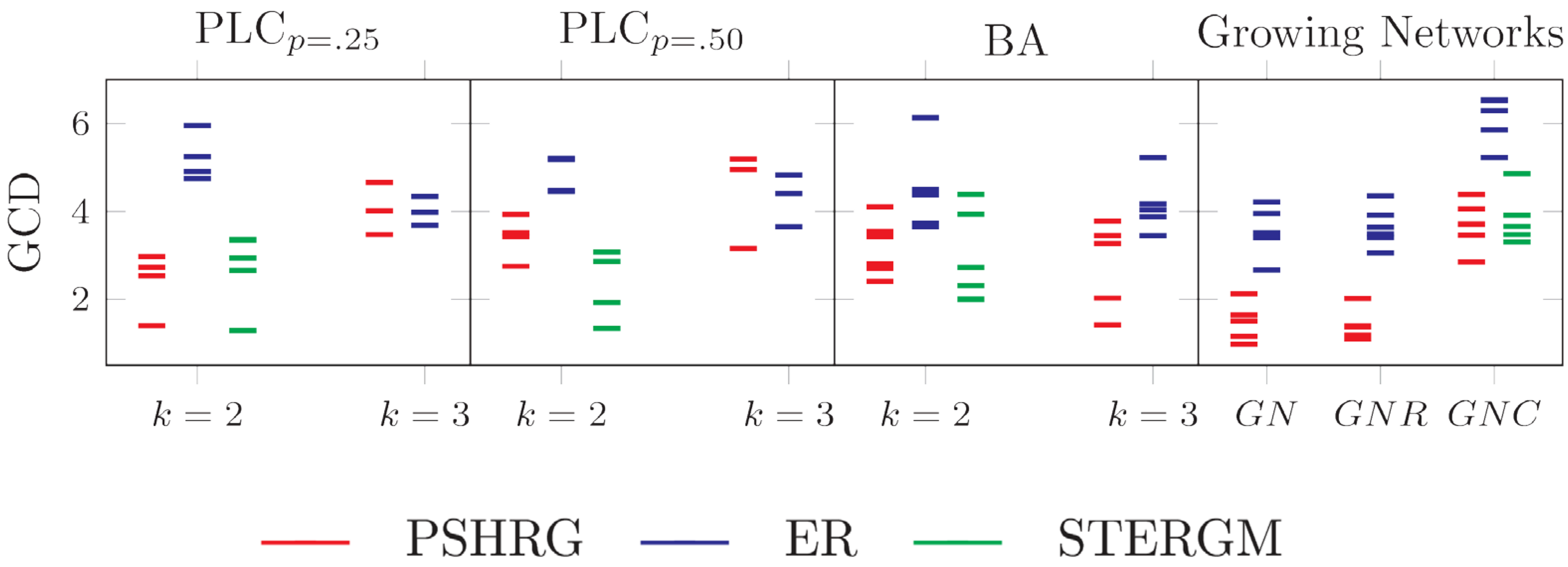
Results: Edge Count

Number of edges generated by ER, STERGM, and PSHRG graph generators for each of the PLC, BA, and Growing Networks (GN, GNR, GNC) graph processes.



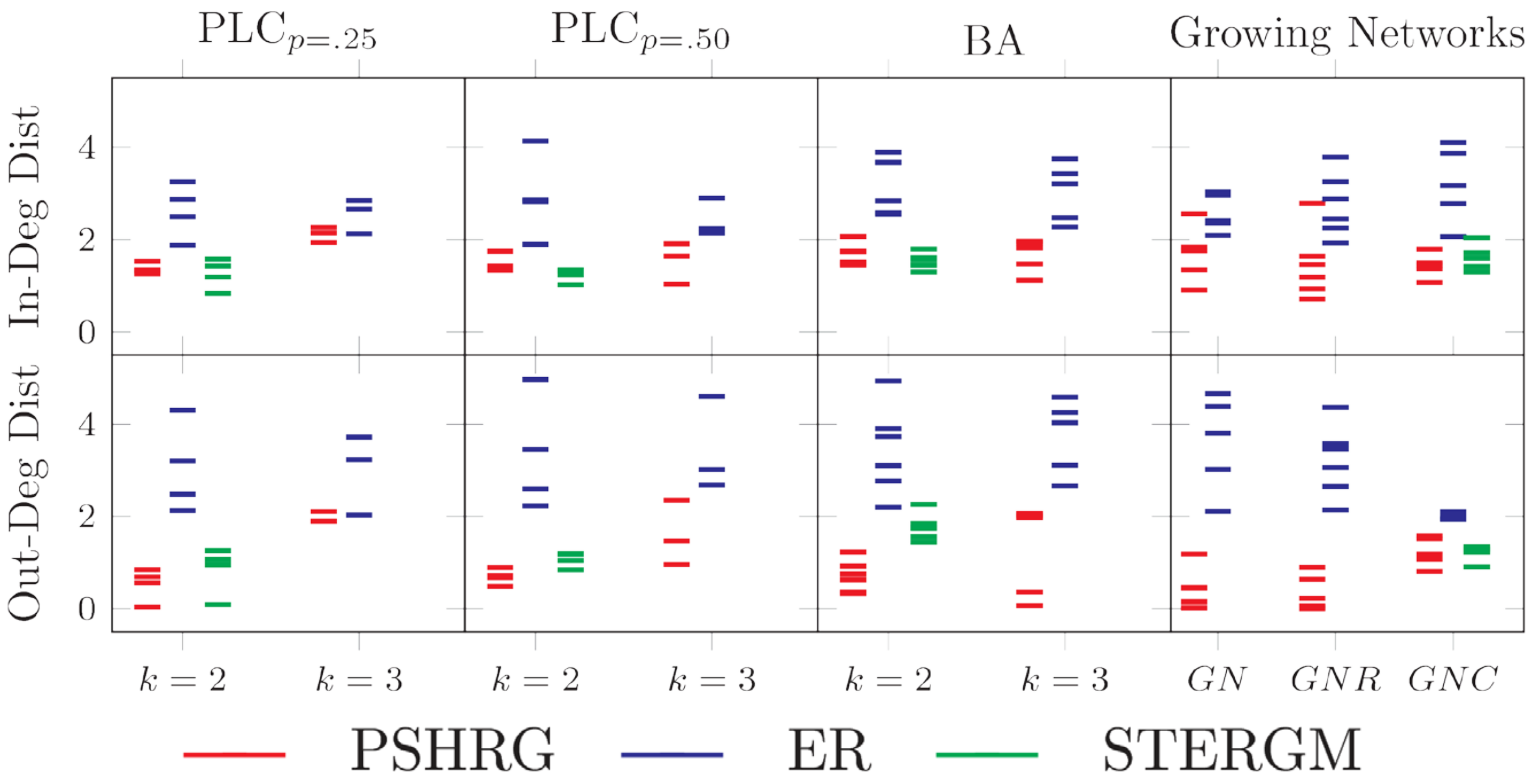
Results: Graphlet Correlation Distance

Graphlet Correlation Distance (GCD). Dashes represent mean GCD scores for various graph sizes (bottom-to-top almost always represents smaller-to-larger graphs), parameters, and models. Lower is better. PSHRG is usually the best.



Results: Degree Distribution

CVM-test statistics of in-degree (top) and out-degree (bottom) distributions for various graph sizes (bottom-to-top almost always represents smaller-to-larger graphs), parameters, and models. Lower is better. PSHRG and STERGM (when available) results are competitive.



Questions?