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Characterisation of Parallel Independence in AGREE-Rewriting

Michael Löwe *(FHDW Hannover)*

ICGT 2018, Toulouse

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Partial Arrow Classifier

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A category has *partial arrow classifiers*, if the following object-indexed family of morphisms exists:

For every object O, there is a monomorphism $\eta_O\colon O \to O^\bullet$ which satisfies the following universal property:

For every pair of morphisms $(i : D \to X, f : D \to O)$ with monic i, there is a unique morphism $(i, f)^\bullet : X \to O^\bullet$ such that the pair (i, f) is pullback of the pair $(\eta_O, (i, f)^\bullet)$.

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O

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$$\begin{tikzcd} O^\bullet \\ O \arrow[u, "\eta_O"'] \end{tikzcd}$$

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$$
\begin{tikzcd}
O^\bullet & X \\
O \arrow[u, "\eta_O"] & D \arrow[l, "f"] \arrow[u, "i"']
\end{tikzcd}
$$

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$$
\begin{tikzcd}
O^\bullet & X \arrow[l, "(i,f)^\bullet"'] \\
O \arrow[u, "\eta_O"] \arrow[ur, phantom, "\text{Pullback}"] & D \arrow[l, "f"] \arrow[u, "i"']
\end{tikzcd}
$$

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$$
\begin{tikzcd}
O^\bullet & X \arrow[l, "{(i,f)^\bullet}"'] \\
O \arrow[u, "\eta_O"] & D \arrow[l, "f"] \arrow[u, "i"']
\end{tikzcd}
$$

Partial arrow

Partial Arrow Classifier

$$
\begin{tikzcd}
O^\bullet \arrow[dr, phantom, "\text{Pullback}"] & X \arrow[l, "(i, f)^\bullet"'] \\
O \arrow[u, "\eta_O"'] & D \arrow[l, "f"] \arrow[u, "i"']
\end{tikzcd}
$$

Partial Arrow Classifier

$$
\begin{tikzcd}
O^\bullet \arrow[dr, phantom, "\text{Pullback}"] & X \arrow[l, "(i,f)^\bullet"'] \\
O \arrow[u, "\eta_O"] \arrow[d, "\mathrm{id}"'] & D \arrow[u, "i"'] \arrow[l, "f"] \arrow[dl, "f"] \\
O &
\end{tikzcd}
$$

Partial Arrow Classifier

$$
\begin{tikzcd}
O^\bullet & & X \arrow[ll, "(i,f)^\bullet"'] \\
O \arrow[u, "\eta_O"] \arrow[d, "\mathrm{id}"'] & \text{Pullback} & D \arrow[ll, "f"] \arrow[u, "i"'] \arrow[dll, "f"] \\
O & &
\end{tikzcd}
$$

Partial Arrow Classifier

$$\begin{tikzcd}
O^{\bullet} & & X \arrow[ll, "(i,f)^{\bullet}"'] \arrow[rr, "\mathrm{id}"] & & X \\
O \arrow[u, "\eta_O"] \arrow[d, "\mathrm{id}"'] \arrow[urr, phantom, "\text{Pullback}"] & & D \arrow[ll, "f"] \arrow[u, "i"'] \arrow[urr, "i"'] \arrow[dll, "f"] \arrow[ur, phantom, "="] \arrow[dl, phantom, "="] & & \\
O & & & &
\end{tikzcd}$$

$$[i, f] = [\eta_O, \mathrm{id}] \circ [\mathrm{id}, (i,f)^{\bullet}] = [\eta_O, \mathrm{id}] \circ \iota\big((i,f)^{\bullet}\big)$$

Partial Arrow Classifier

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$$
\begin{tikzcd}
O^\bullet & X \arrow[l, "(i,f)^\bullet"'] \arrow[r, "\mathrm{id}"] & X \\
O \arrow[u, "\eta_O"] \arrow[d, "\mathrm{id}"'] & D \arrow[l, "f"] \arrow[u, "i"'] \arrow[ur, "i"'] \arrow[dl, "f"] \arrow[ul, phantom, "\text{Pullback}"] & \\
O & &
\end{tikzcd}
$$

$$[i,f] = [\eta_O, \mathrm{id}] \circ [\mathrm{id}, (i,f)^\bullet] = [\eta_O, \mathrm{id}] \circ \iota\left((i,f)^\bullet\right)$$

Partial Arrow Classifier

A category has *partial arrow classifiers*, if for every object O

there is object $O^\bullet$ and *partial morphism* $\varepsilon_O\colon O^\bullet \rightarrow O$ such that

for every object X and *partial morphism* $(p : X \rightarrow O)$

there is *unique total morphism* $p^\bullet : X \rightarrow O^\bullet$ with $\varepsilon_O \circ \iota(p^\bullet) = p$,

where functor ι is given by: $\iota_O : O \longmapsto O$ and $\iota_M : m \longmapsto [\mathrm{id}, m]$.

The embedding from category C (with *total* arrows) into the category of *partial* arrows over C is a ***free construction***!

Partial Arrow Classifier

Pushouts are hereditary

Pushouts preserve monomorphism

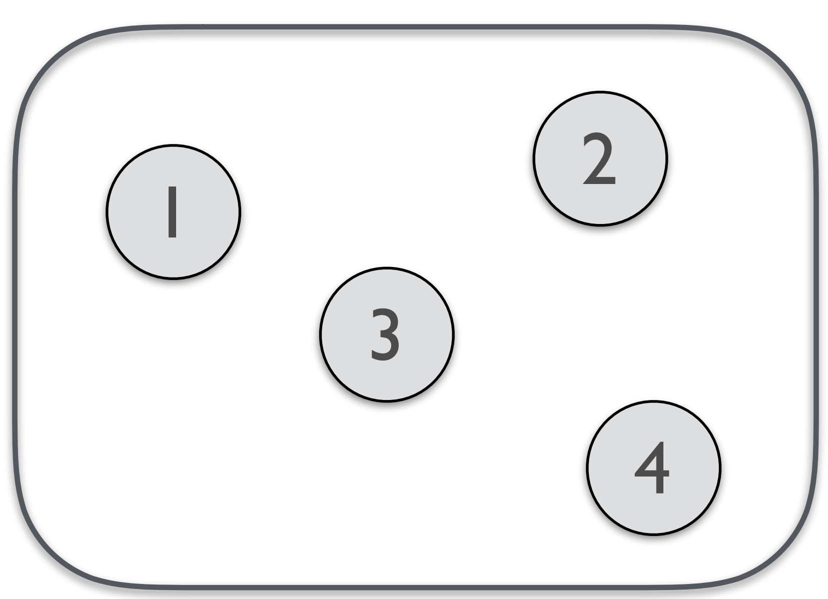
Pushouts along monomorphisms are pullbacks

Category has epi-mono-factorisation

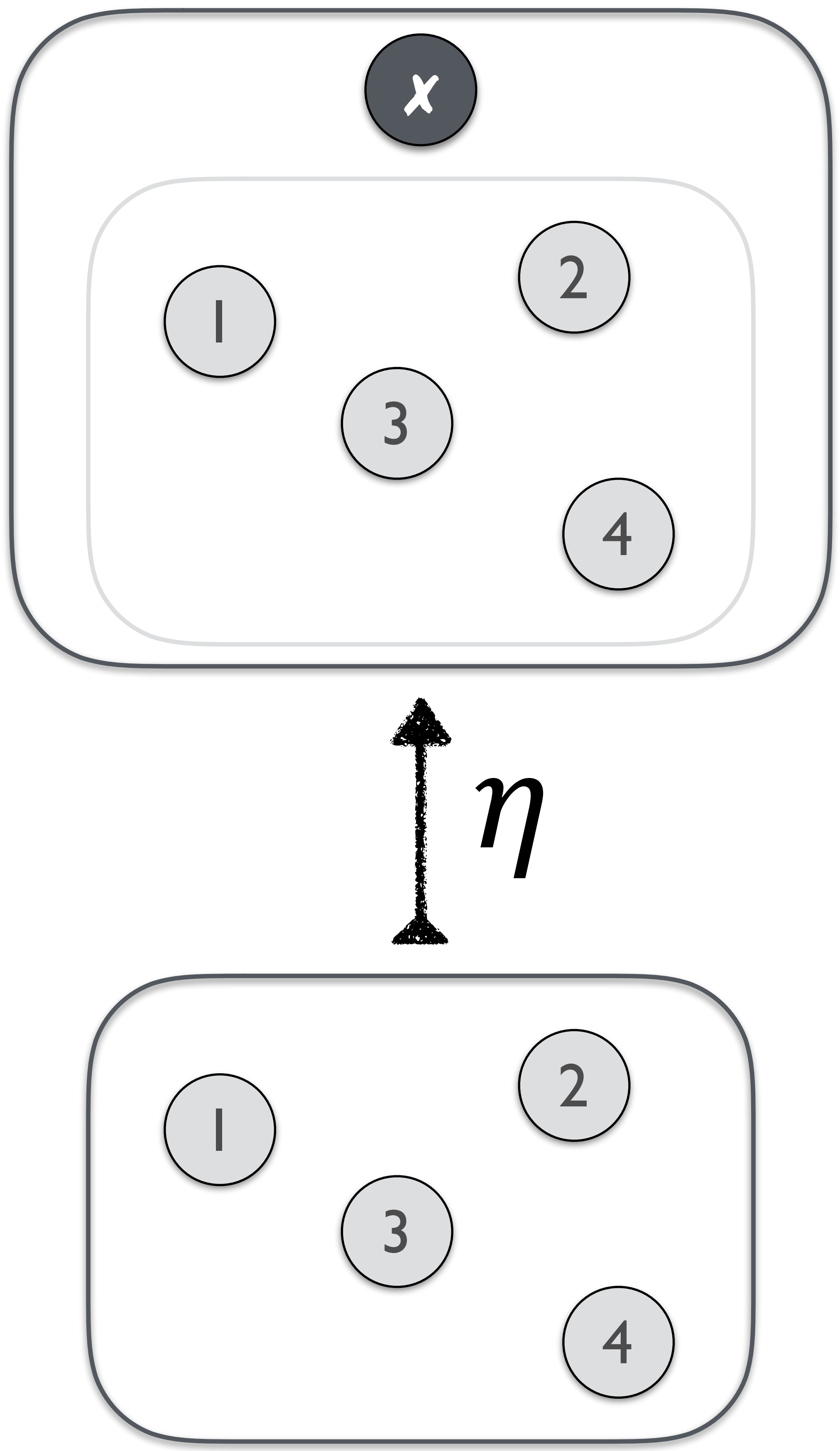
Pullbacks are preserved by embedding ……

The embedding from category *C* (with *total* arrows) into the category of *partial* arrows over *C* is a ***free construction***!

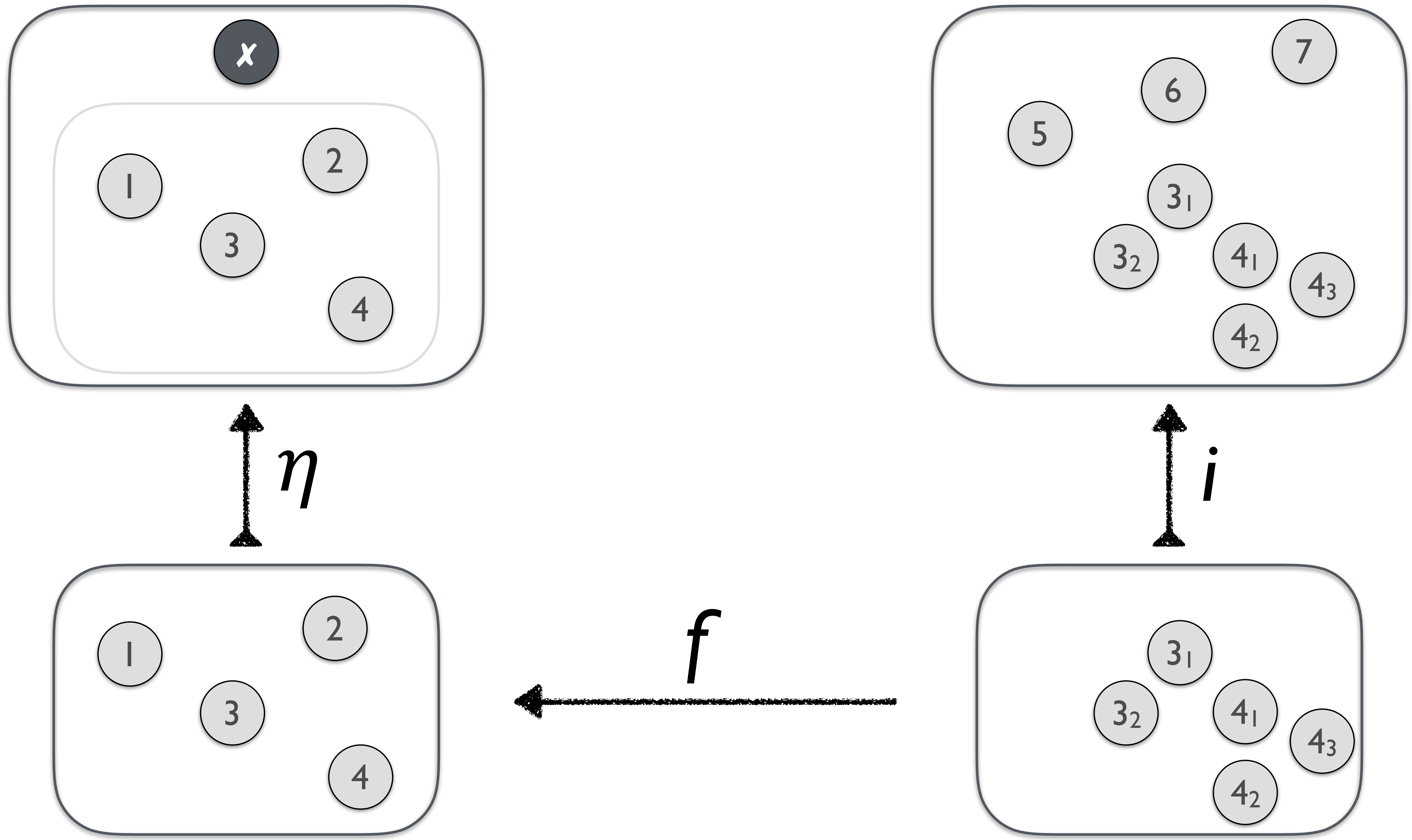
Partial Arrow Classifier: Set



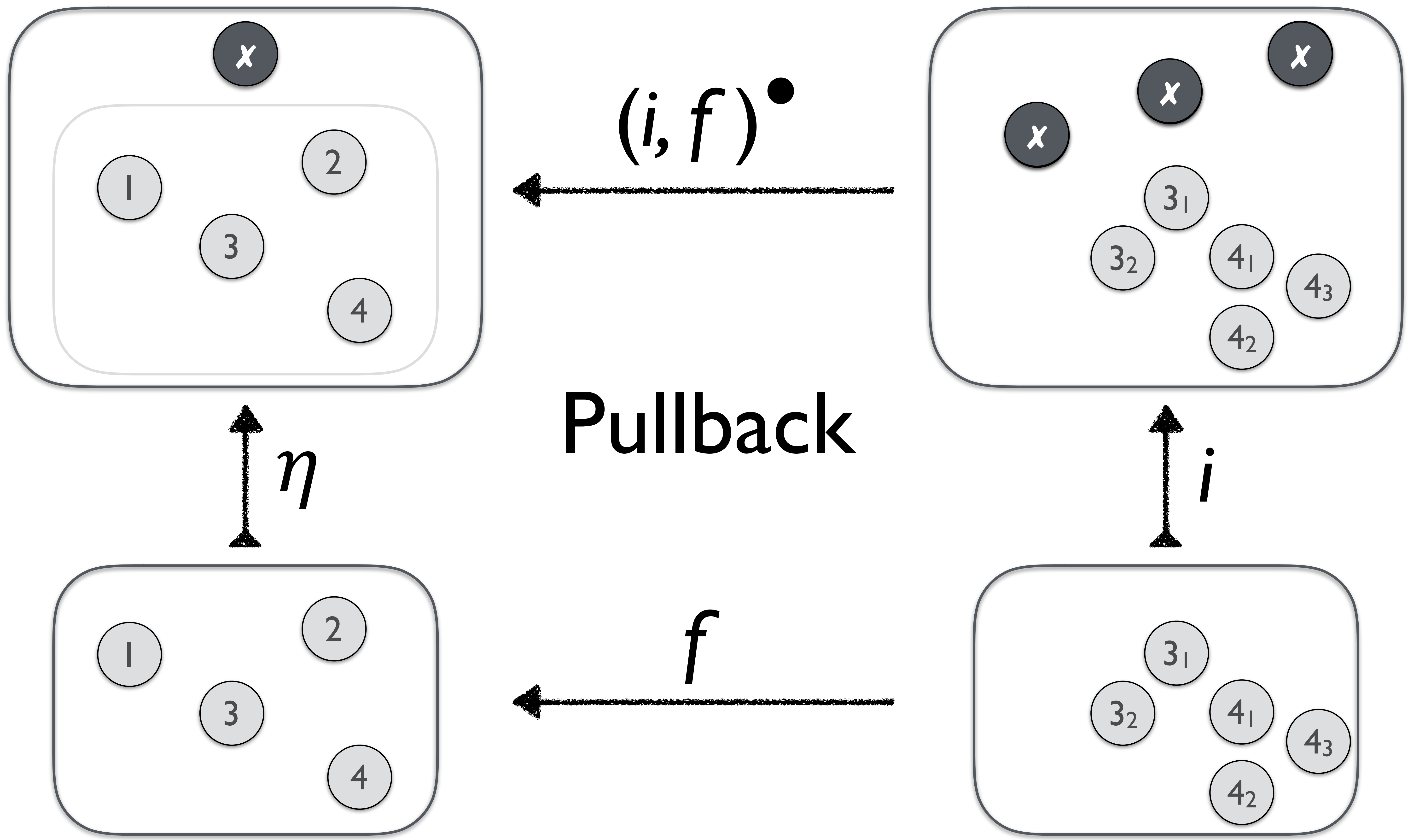
Partial Arrow Classifier: Set



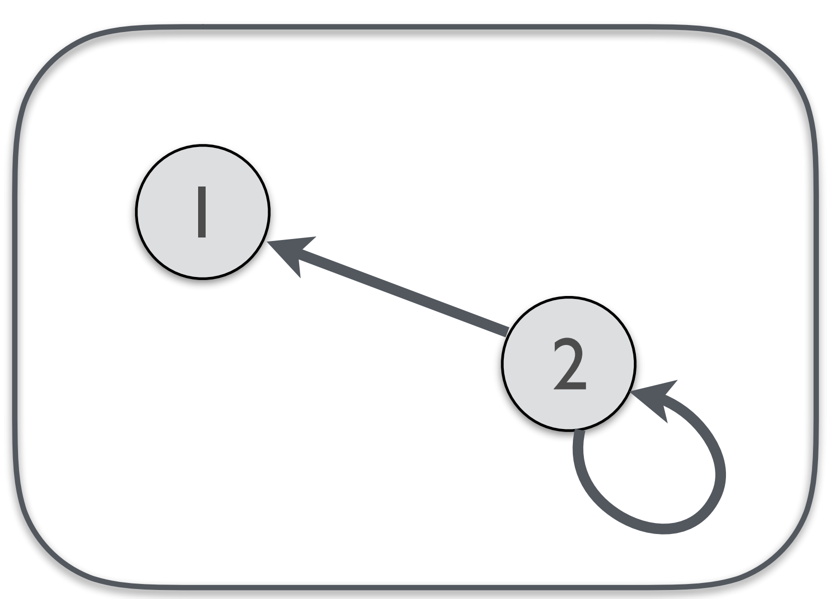
Partial Arrow Classifier: Set



Partial Arrow Classifier: Set



Partial Arrow Classifier: Graph



Partial Arrow Classifier: Graph

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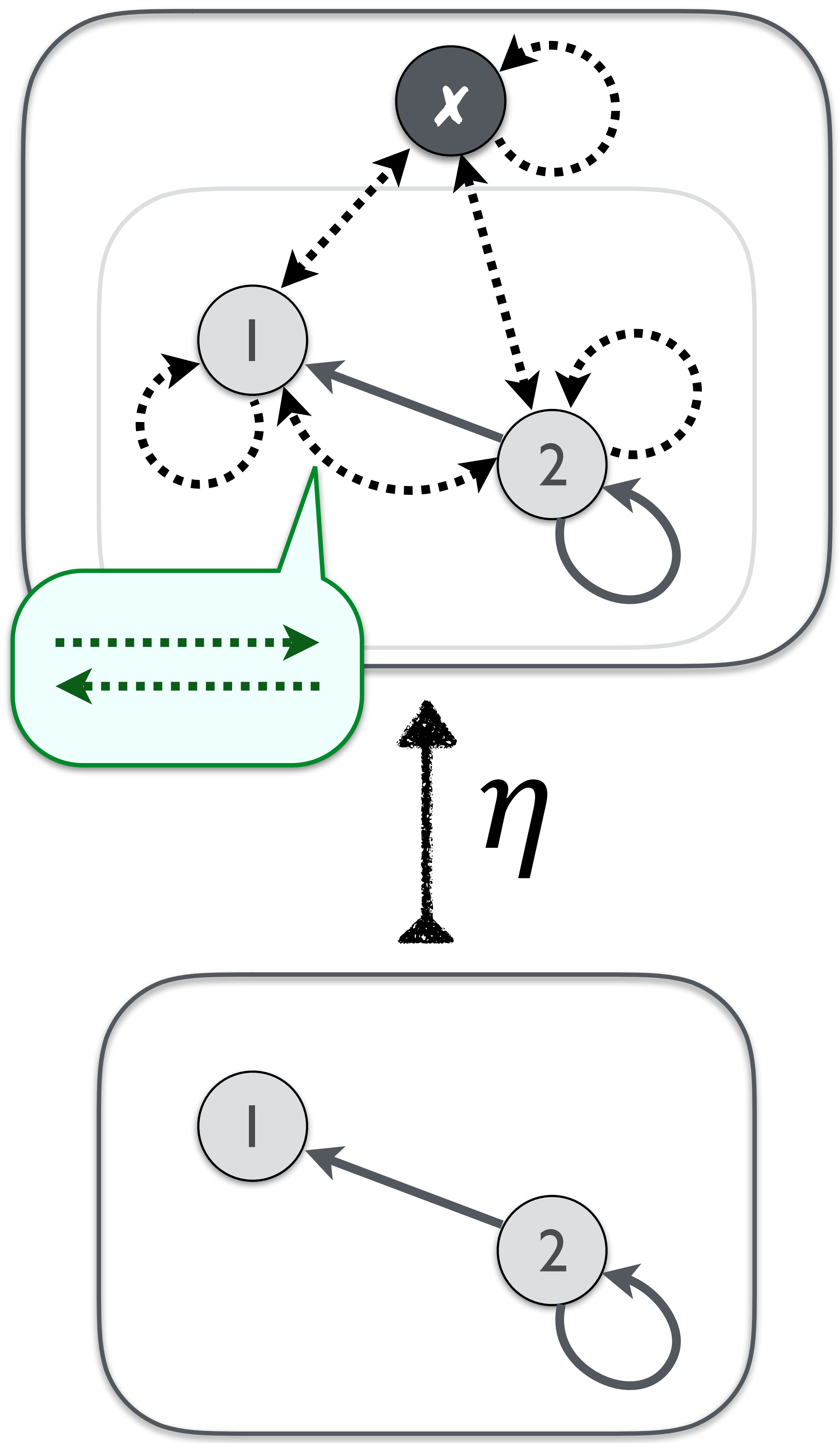

Partial Arrow Classifier: Graph

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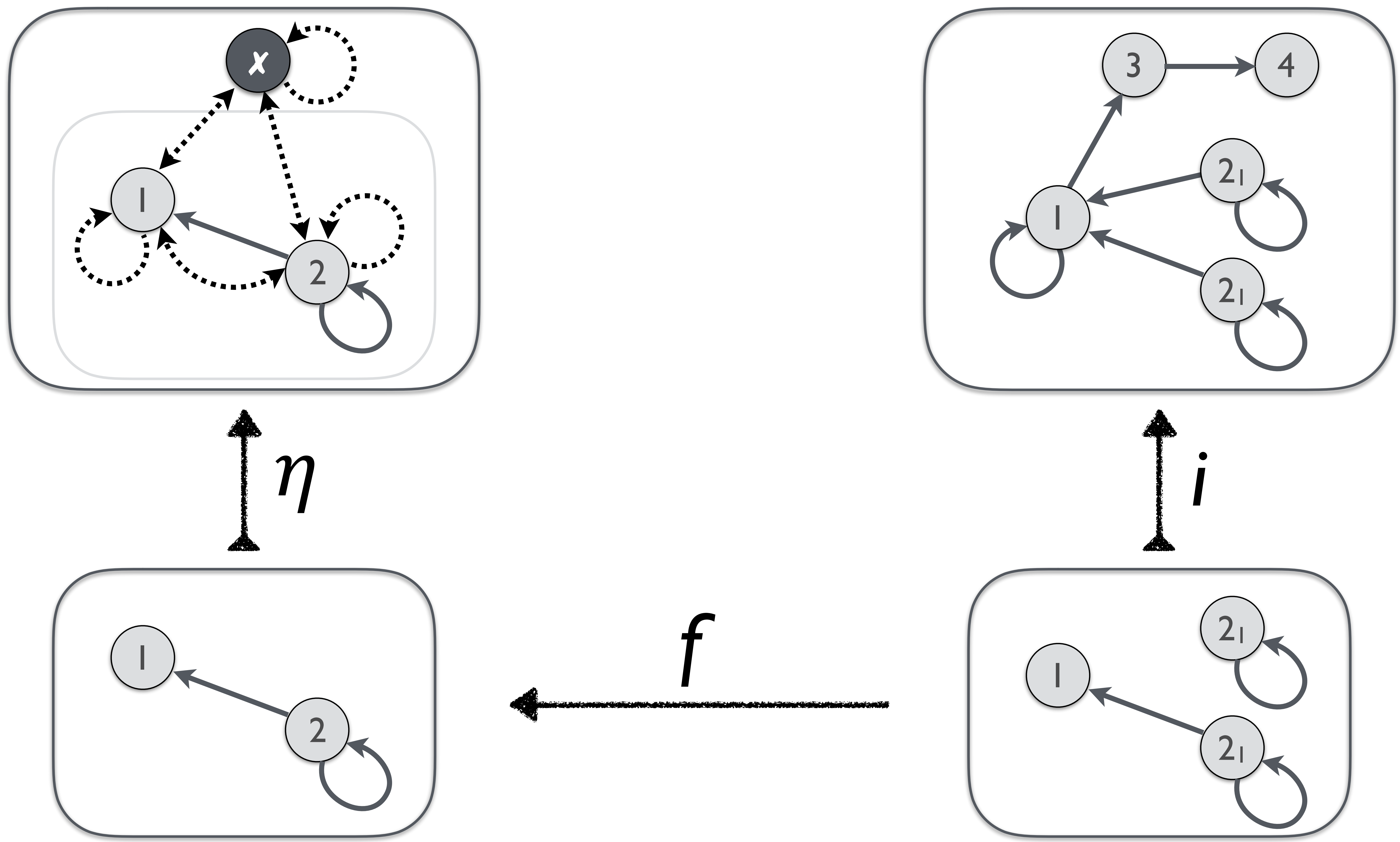
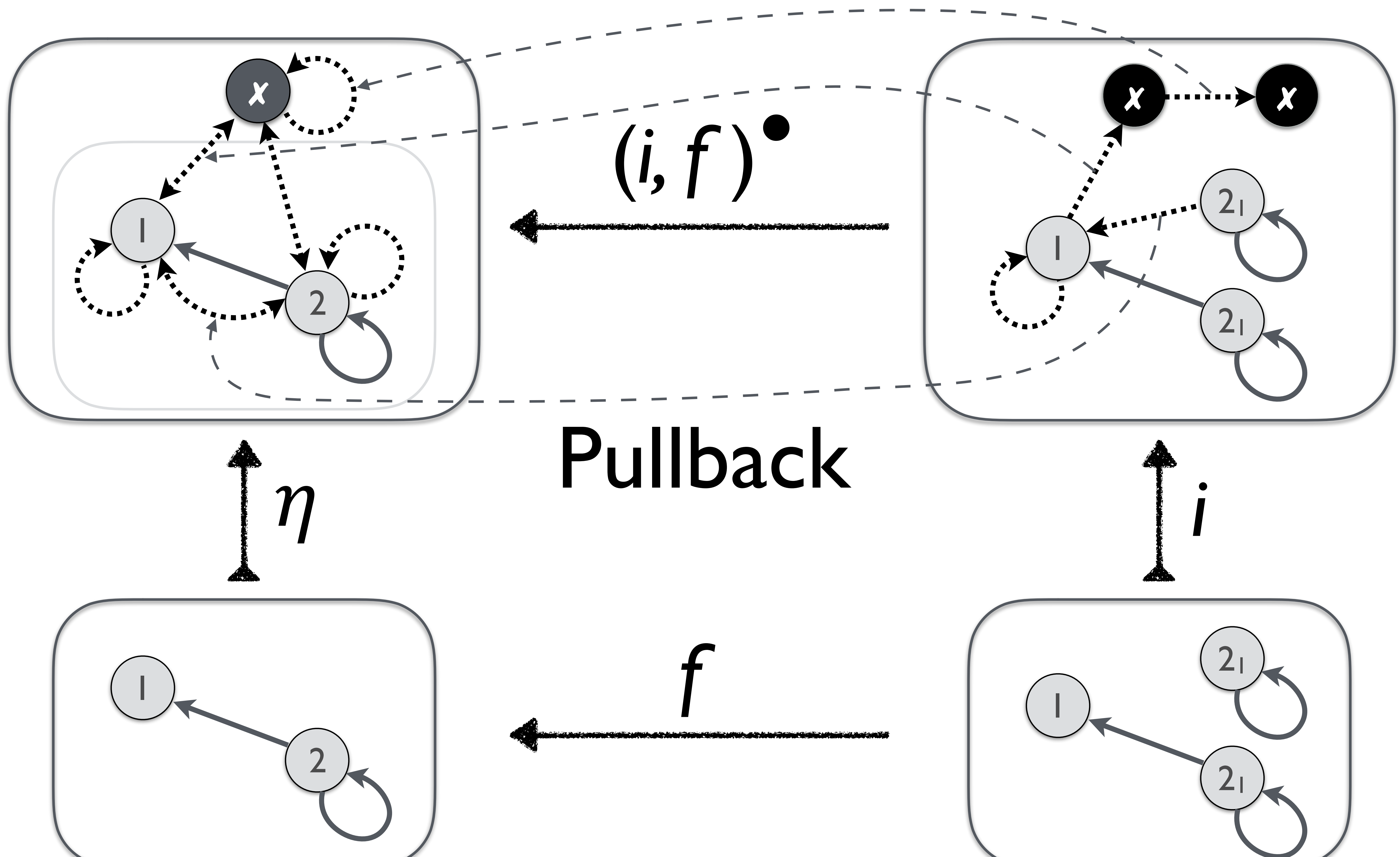
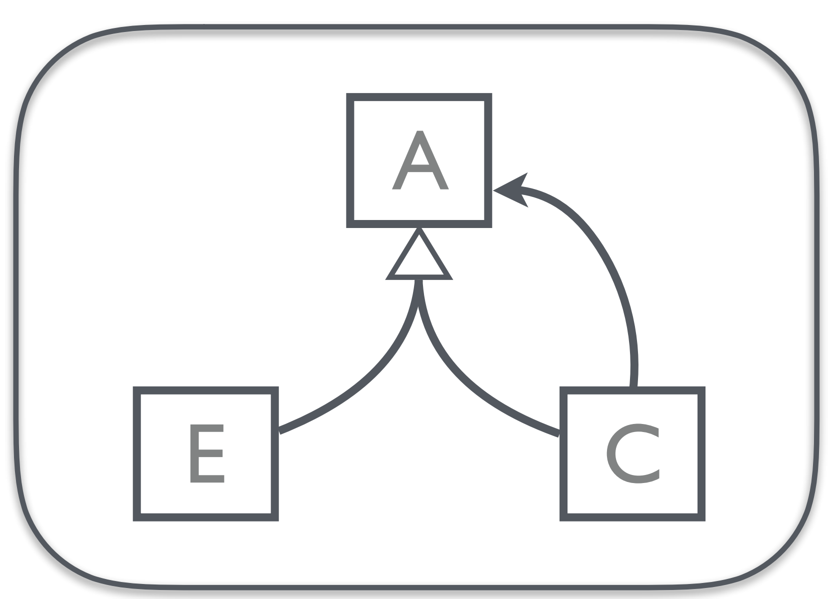


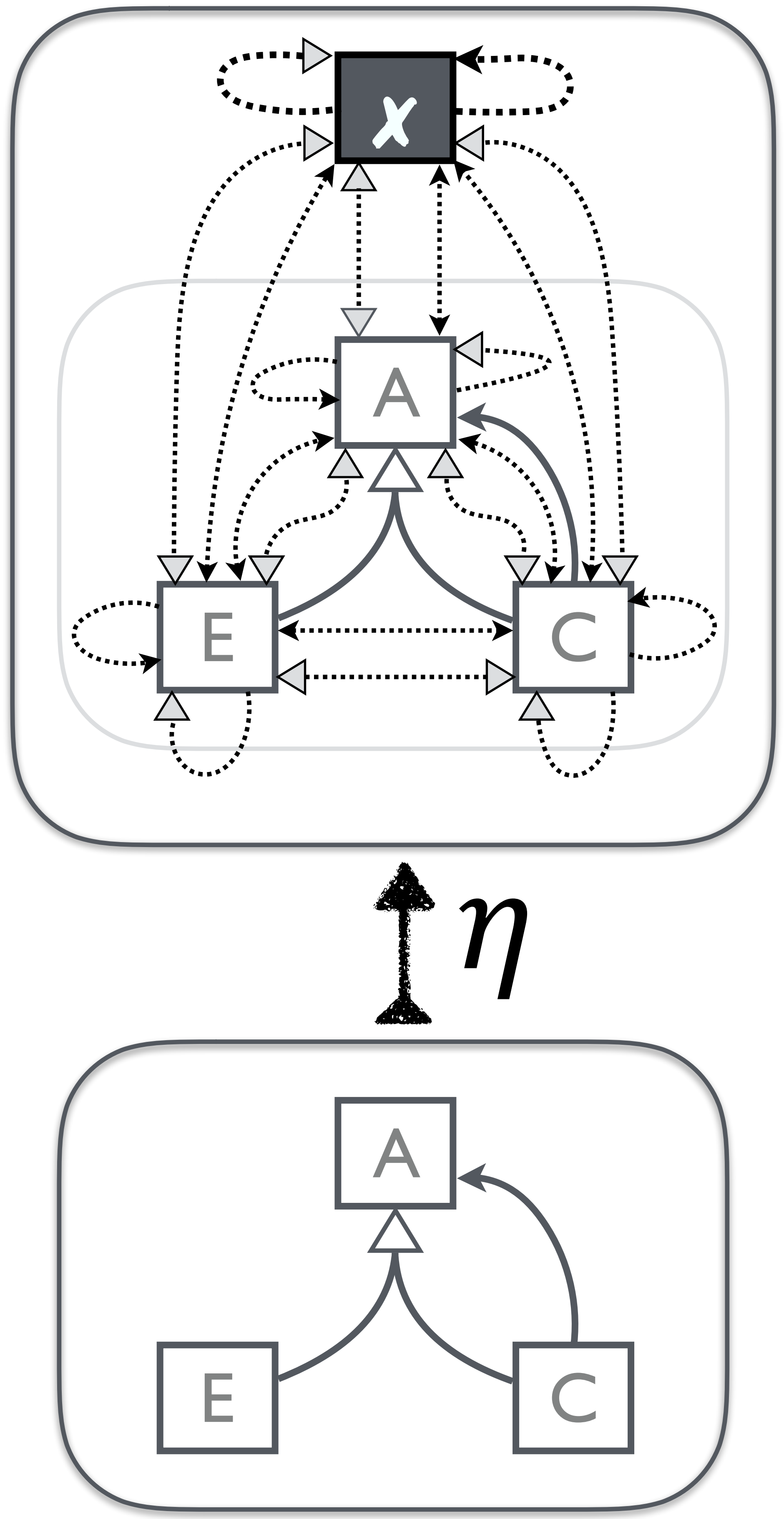
Partial Arrow Classifier: Graph



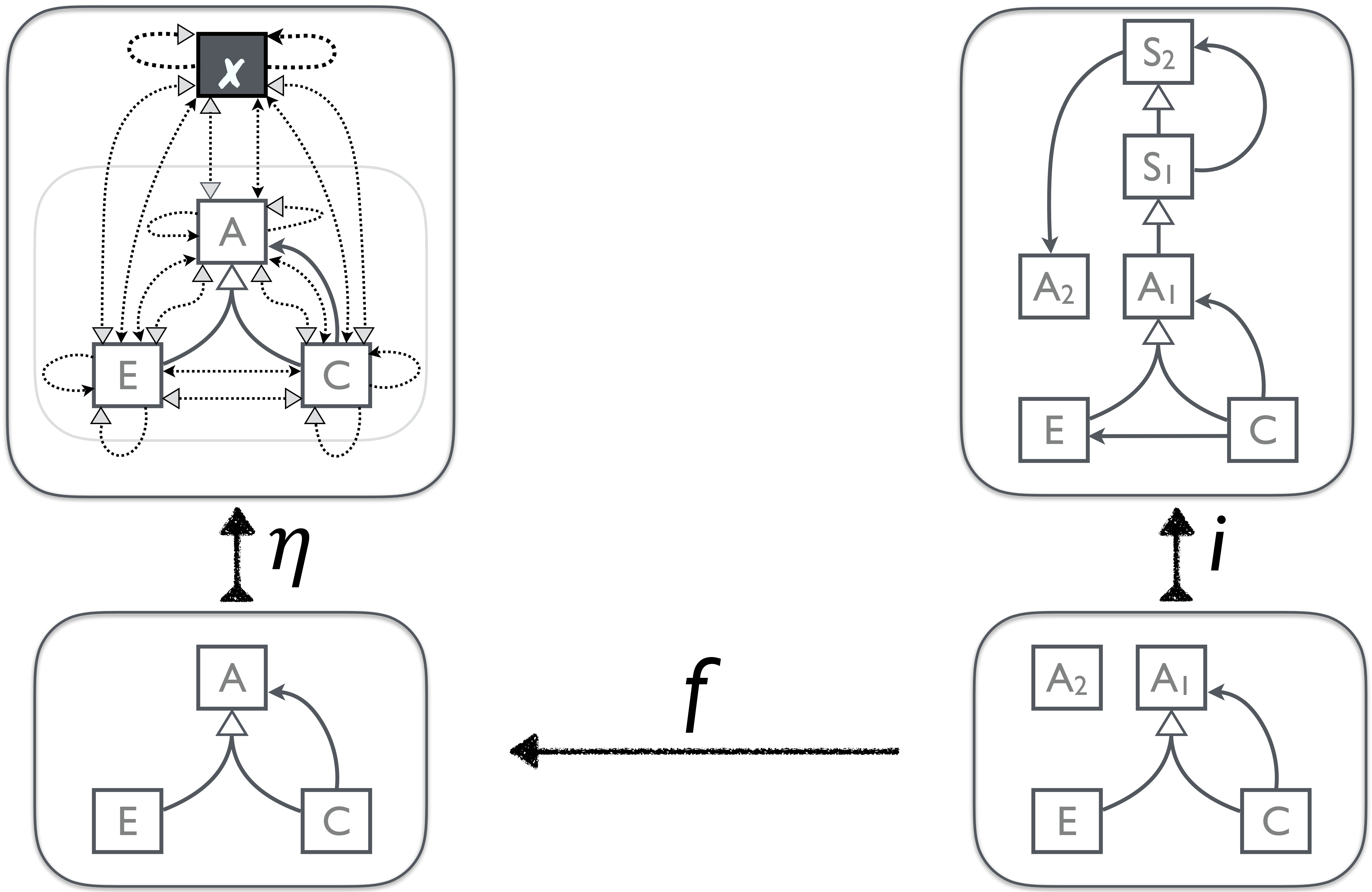
Partial Arrow Classifier: OO-Model



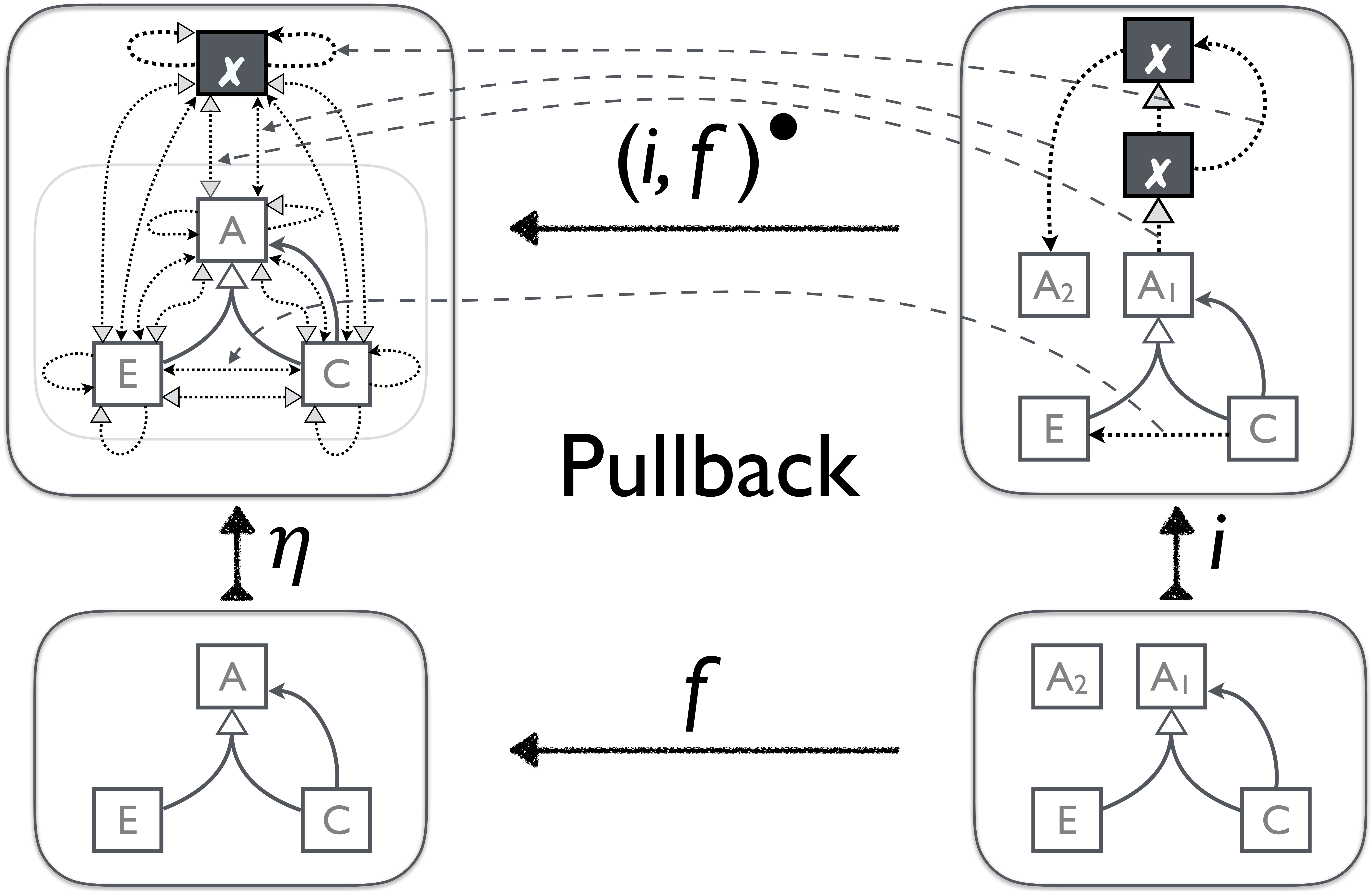
Partial Arrow Classifier: OO-Model



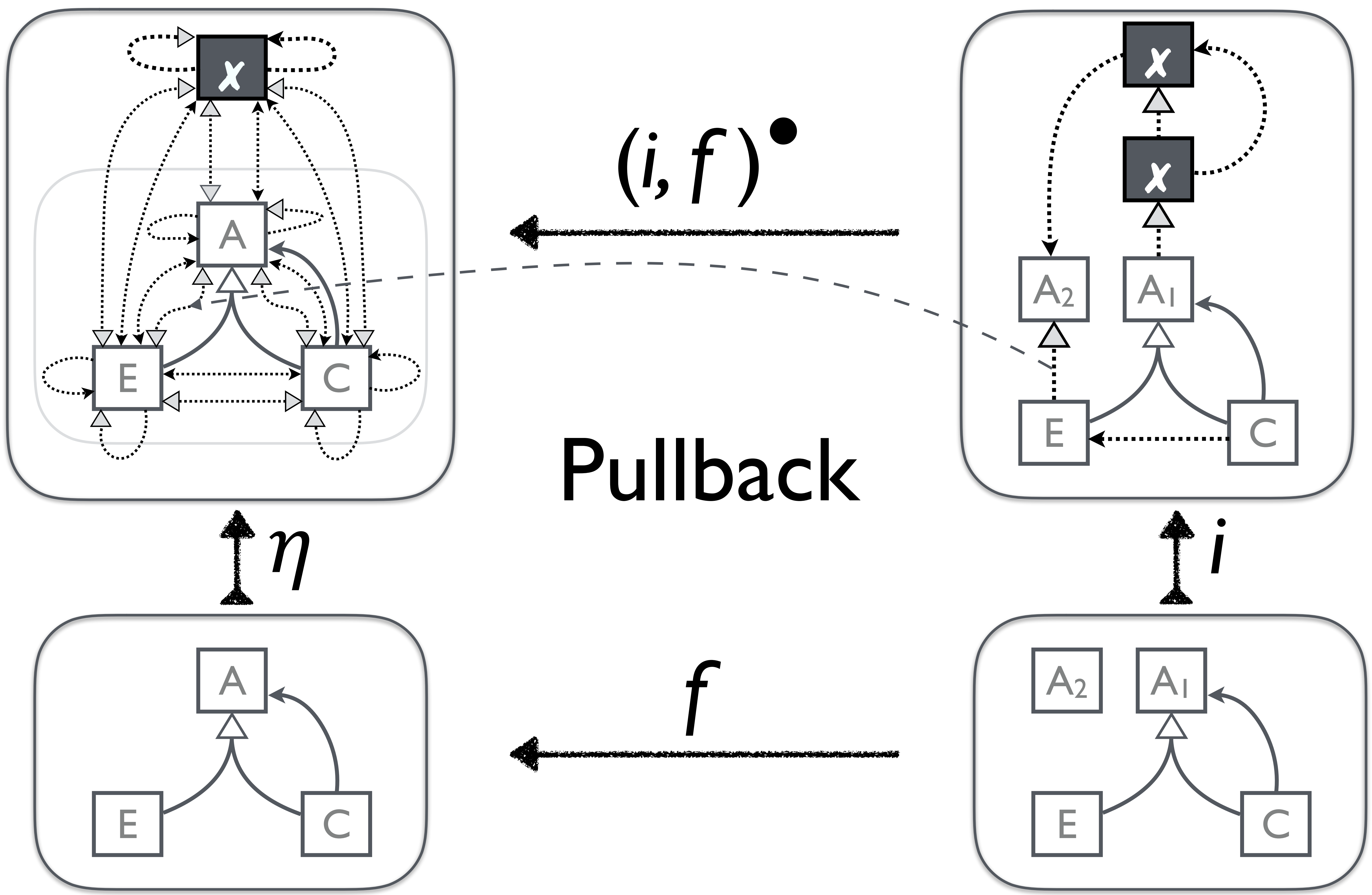
Partial Arrow Classifier: OO-Model



Partial Arrow Classifier: OO-Model



Partial Arrow Classifier: OO-Model



‚Inversion‘ of Matches

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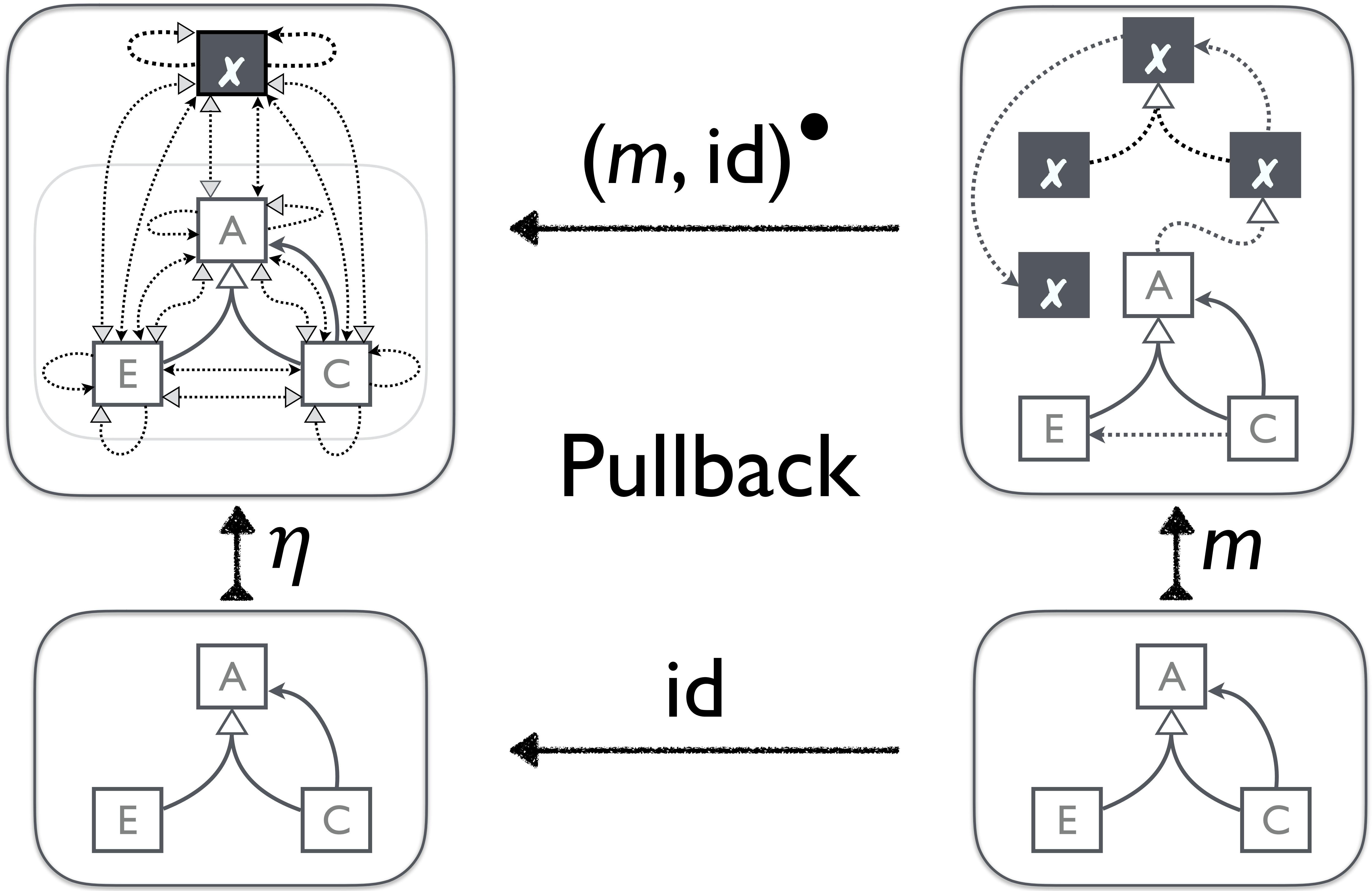

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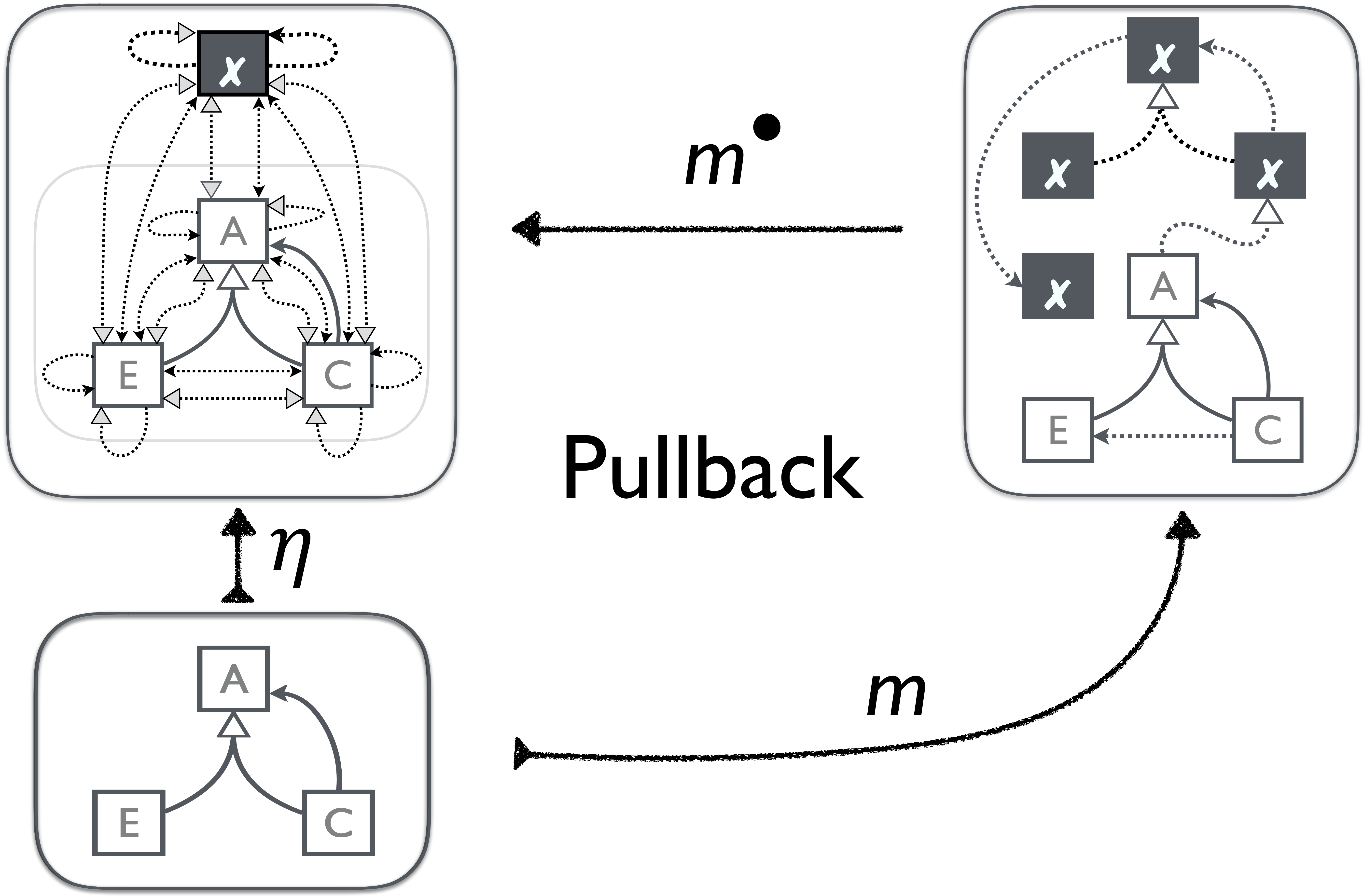

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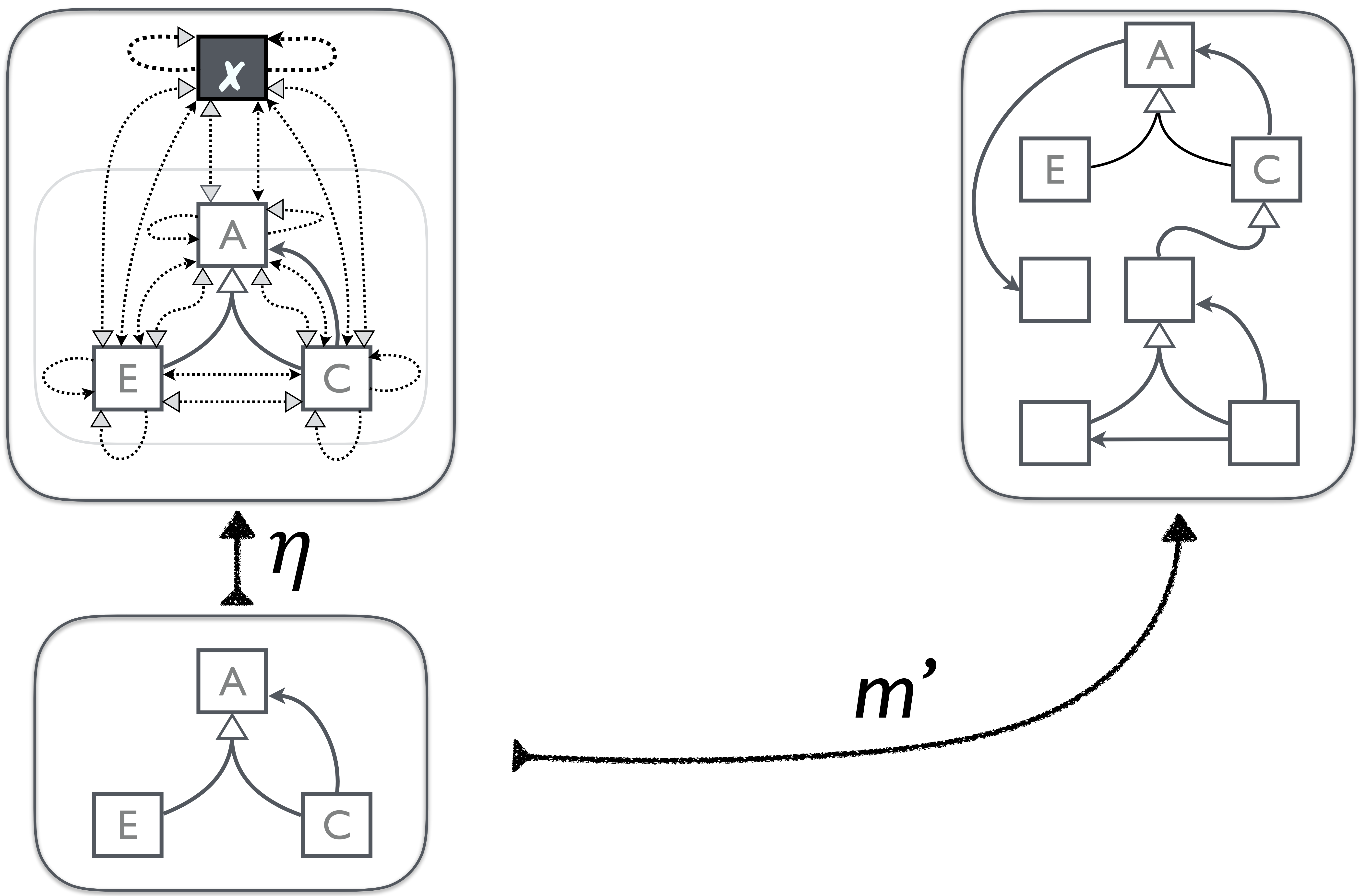

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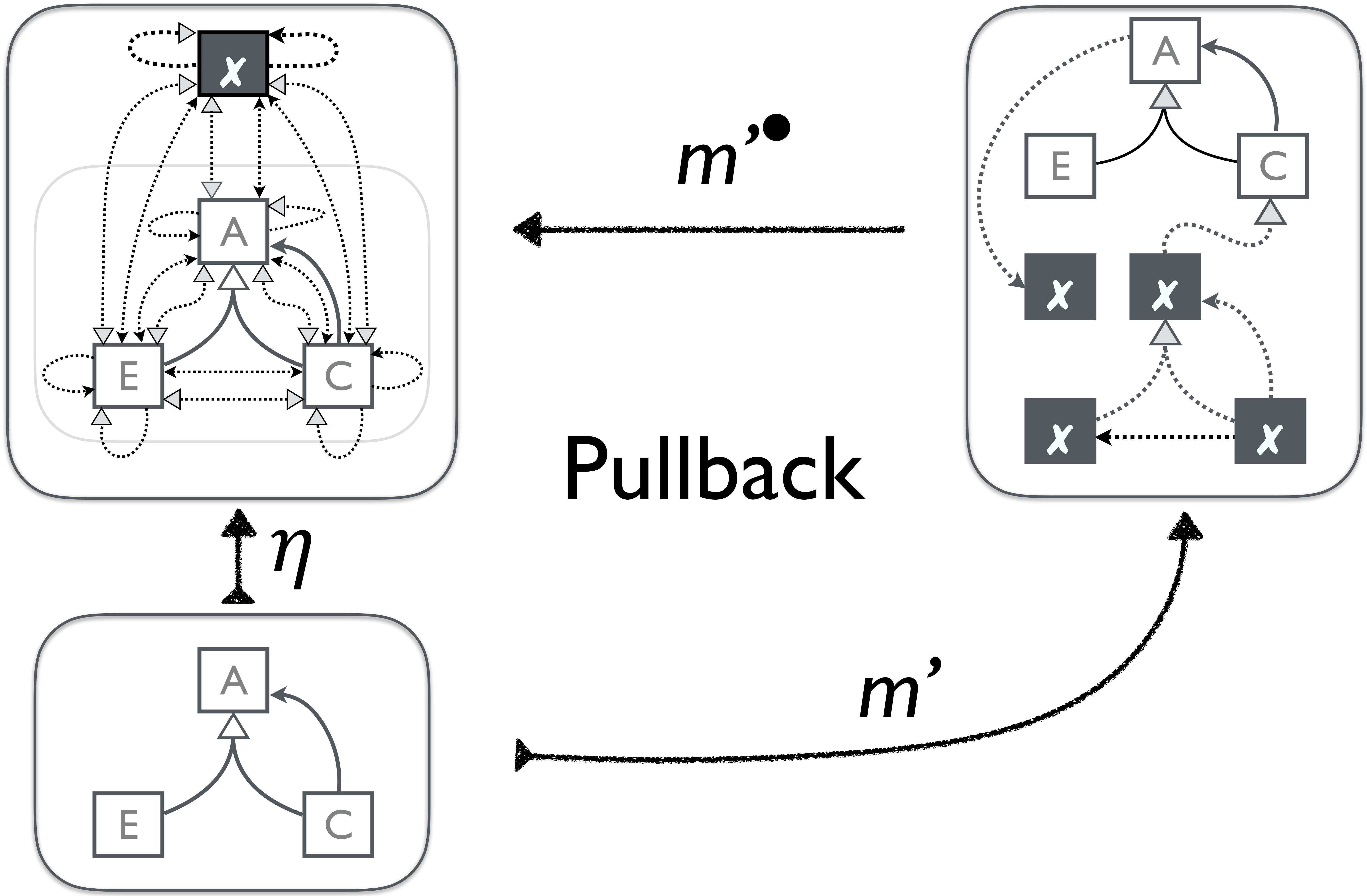


AGREE-Rewriting

AGREE-Rewriting

Rule:

$$\begin{tikzcd} & T & \\ L & K \arrow[l, "l"'] \arrow[r, "r"] \arrow[u, "t"'] & R \end{tikzcd}$$

AGREE-Rewriting

Rule:

$$\begin{tikzcd} L^\bullet \arrow[dr, phantom, "(PB)"] & T \arrow[l, "(t,\, l)^\bullet"'] & \\ L \arrow[u, "\eta_L"'] & K \arrow[l, "l"'] \arrow[u, "t"'] \arrow[r, "r"] & R \end{tikzcd}$$

AGREE-Rewriting

Inverse Match:

Rule:

Base Match:

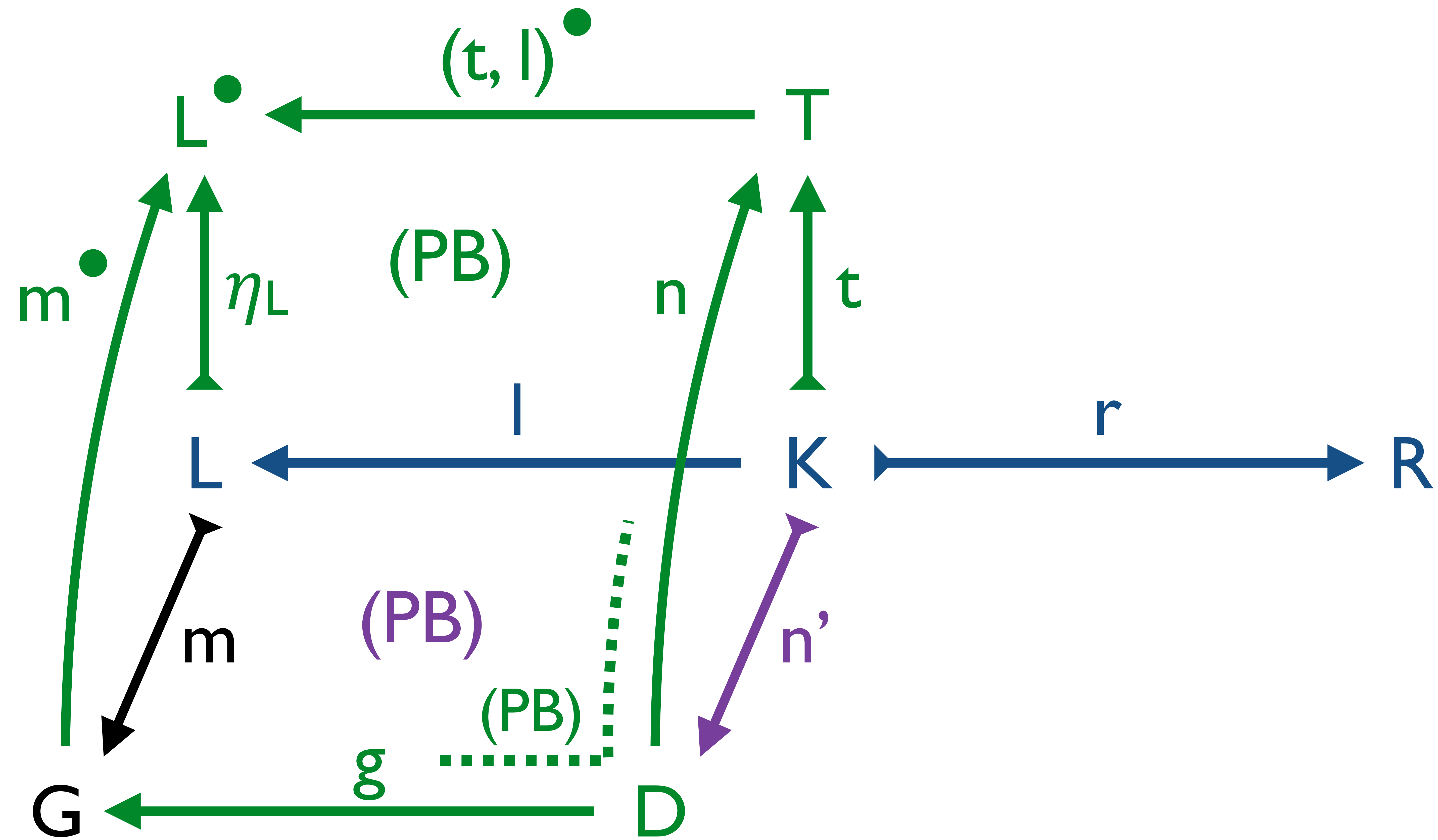
$$
\begin{tikzcd}
 & L^\bullet \arrow[dr, phantom, "(PB)"] & T \arrow[l, "(t, l)^\bullet"'] & \\
 & L \arrow[u, "\eta_L"'] \arrow[dl, "m"] & K \arrow[l, "l"'] \arrow[u, "t"'] \arrow[r, "r"] & R \\
G \arrow[uur, bend left, "m^\bullet"] & & &
\end{tikzcd}
$$

AGREE-Rewriting

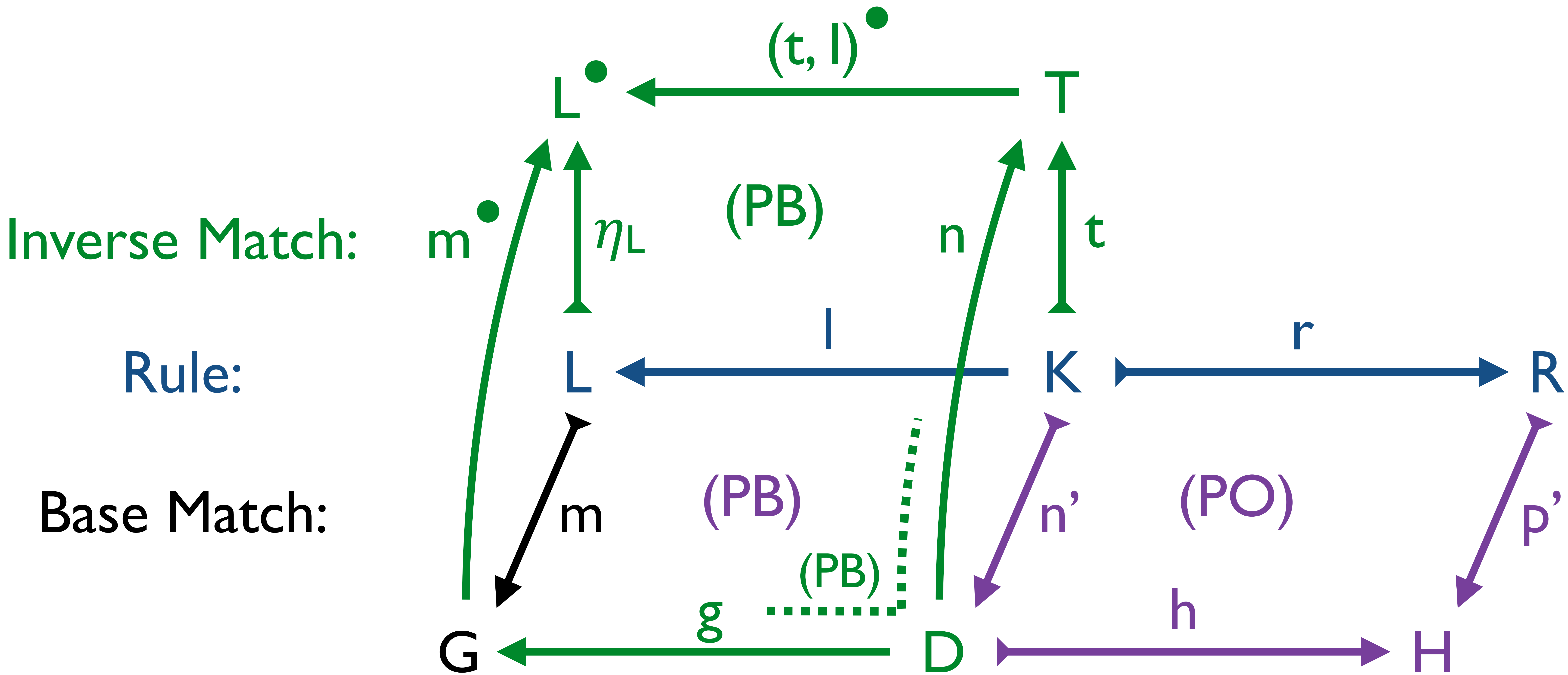
Inverse Match:

Rule:

Base Match:



AGREE-Rewriting



AGREE-Rewriting

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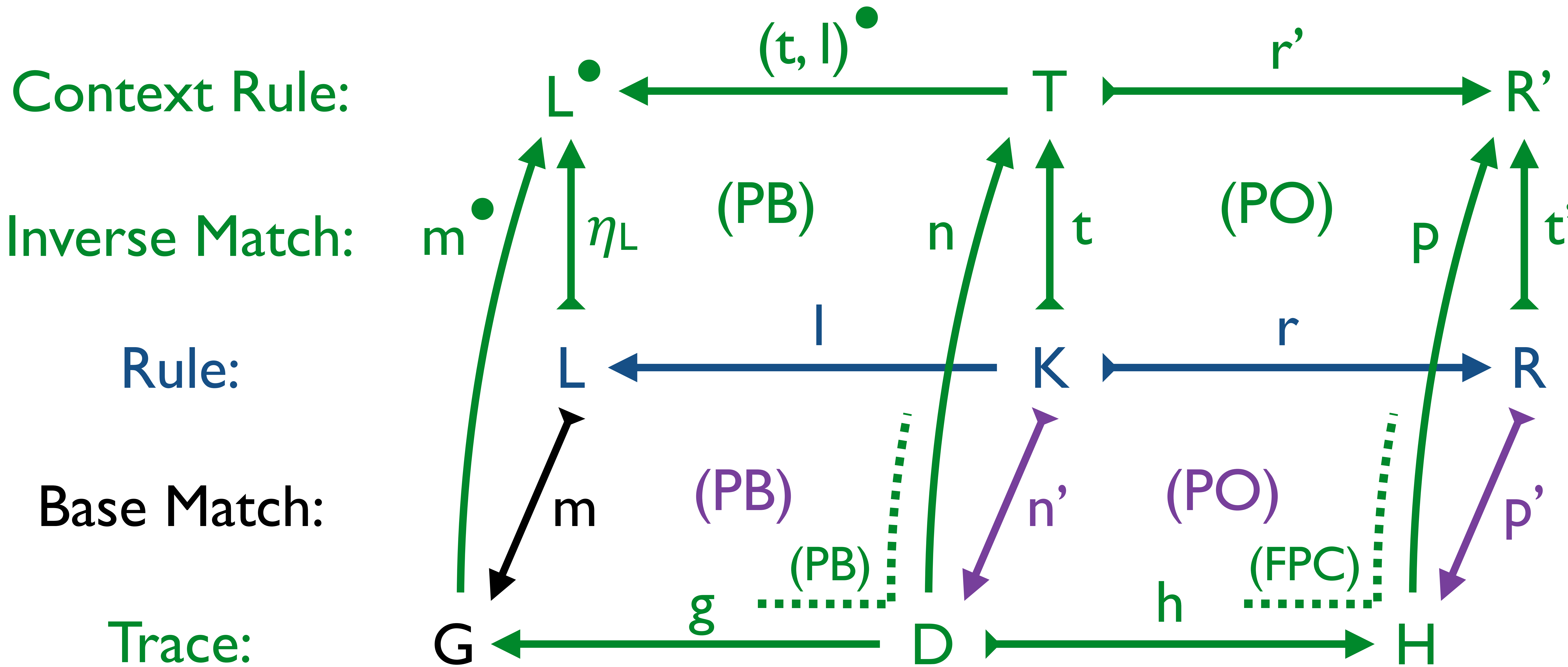

AGREE: Practical Example

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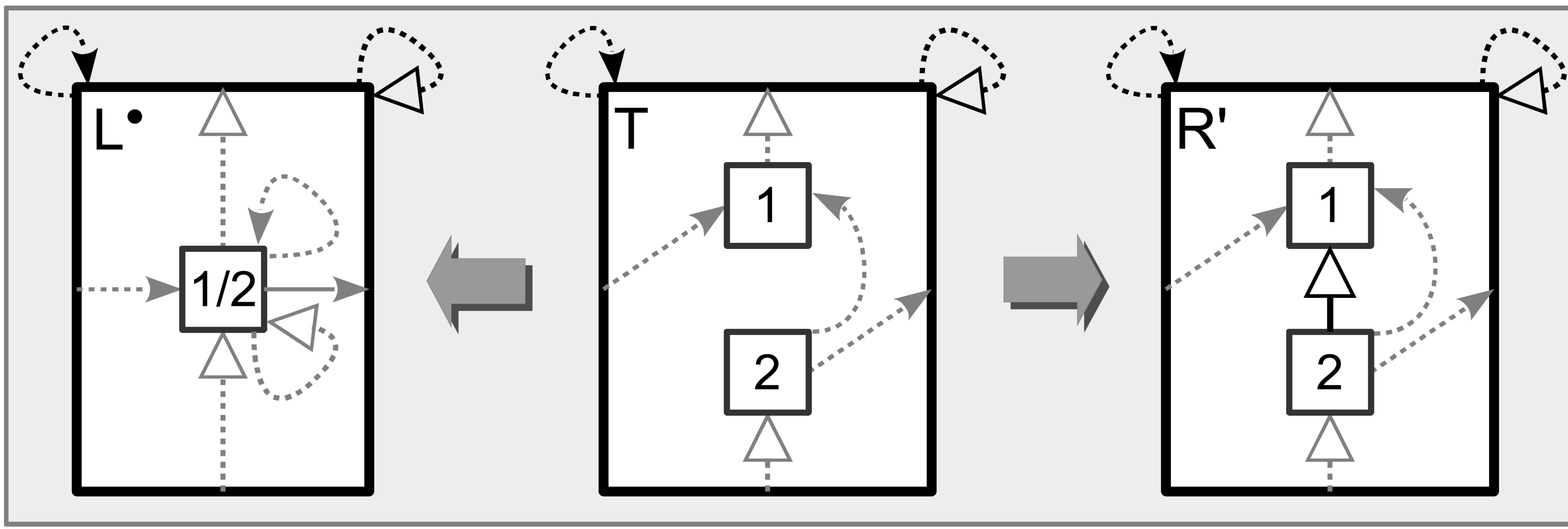
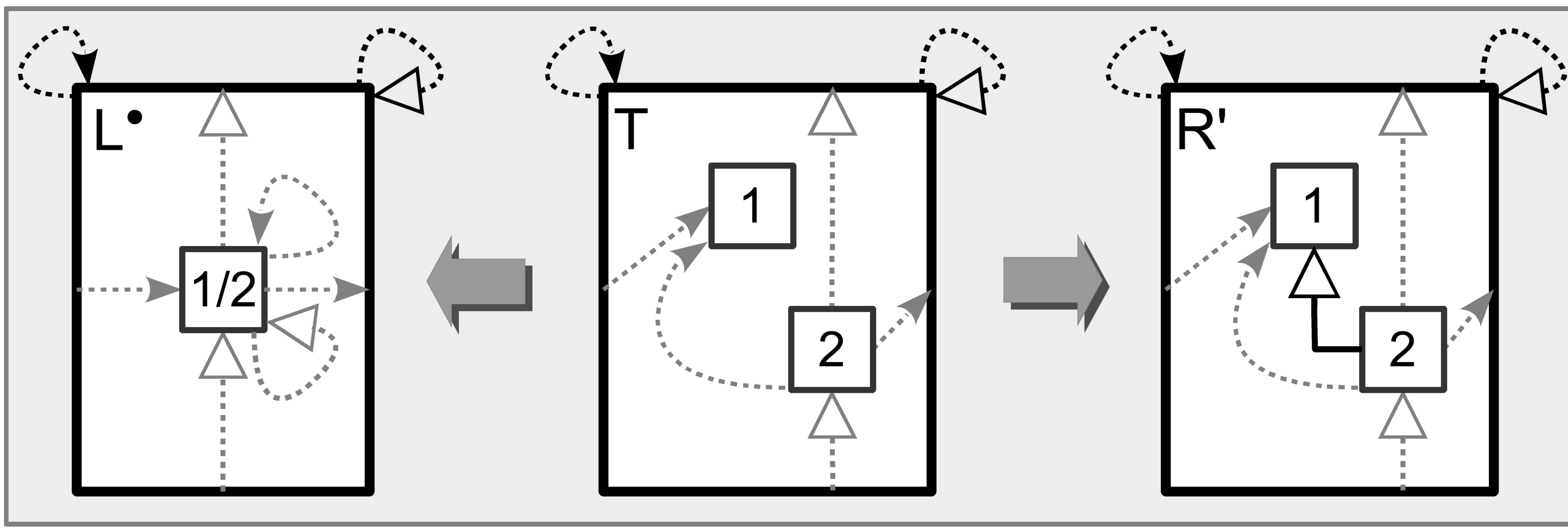
Extract abstract type (Version 1):

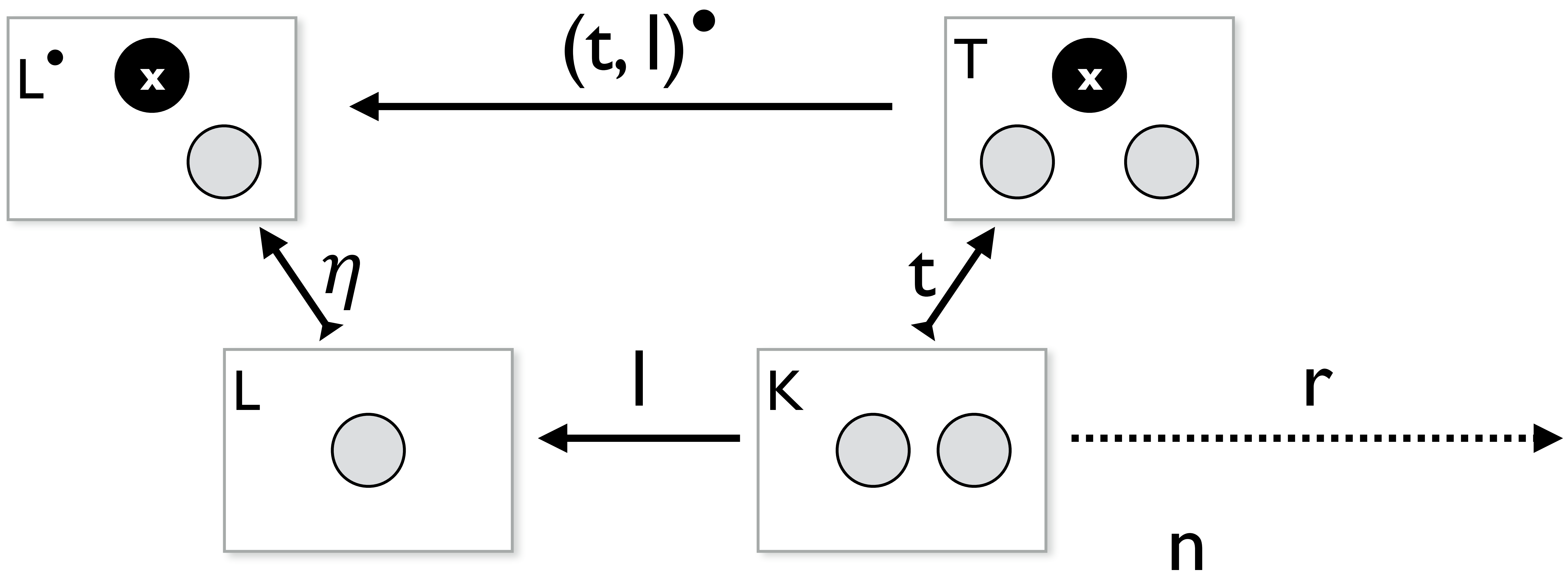


AGREE: Practical Example

Extract abstract type (Version 2):



AGREE: Local Copies



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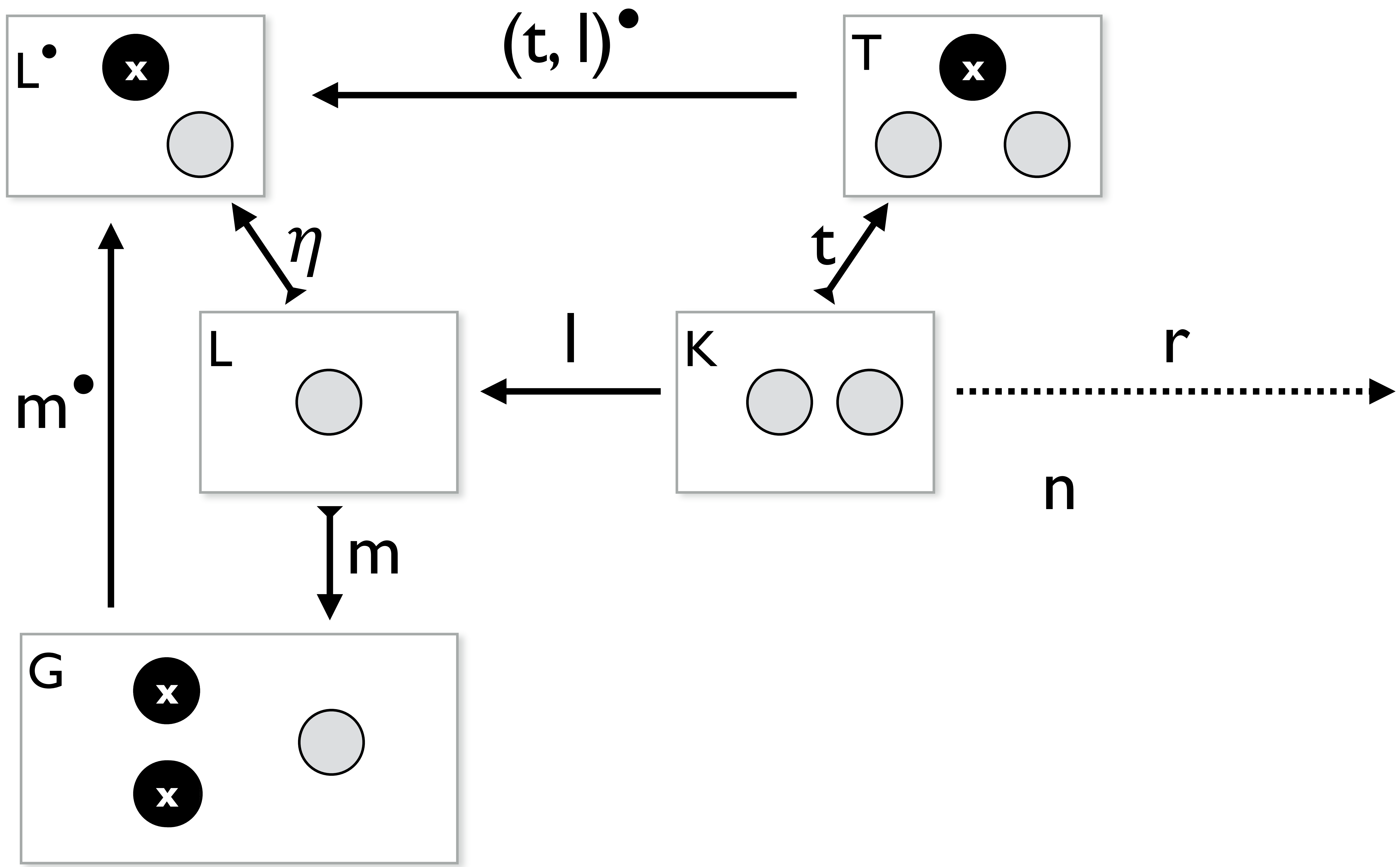
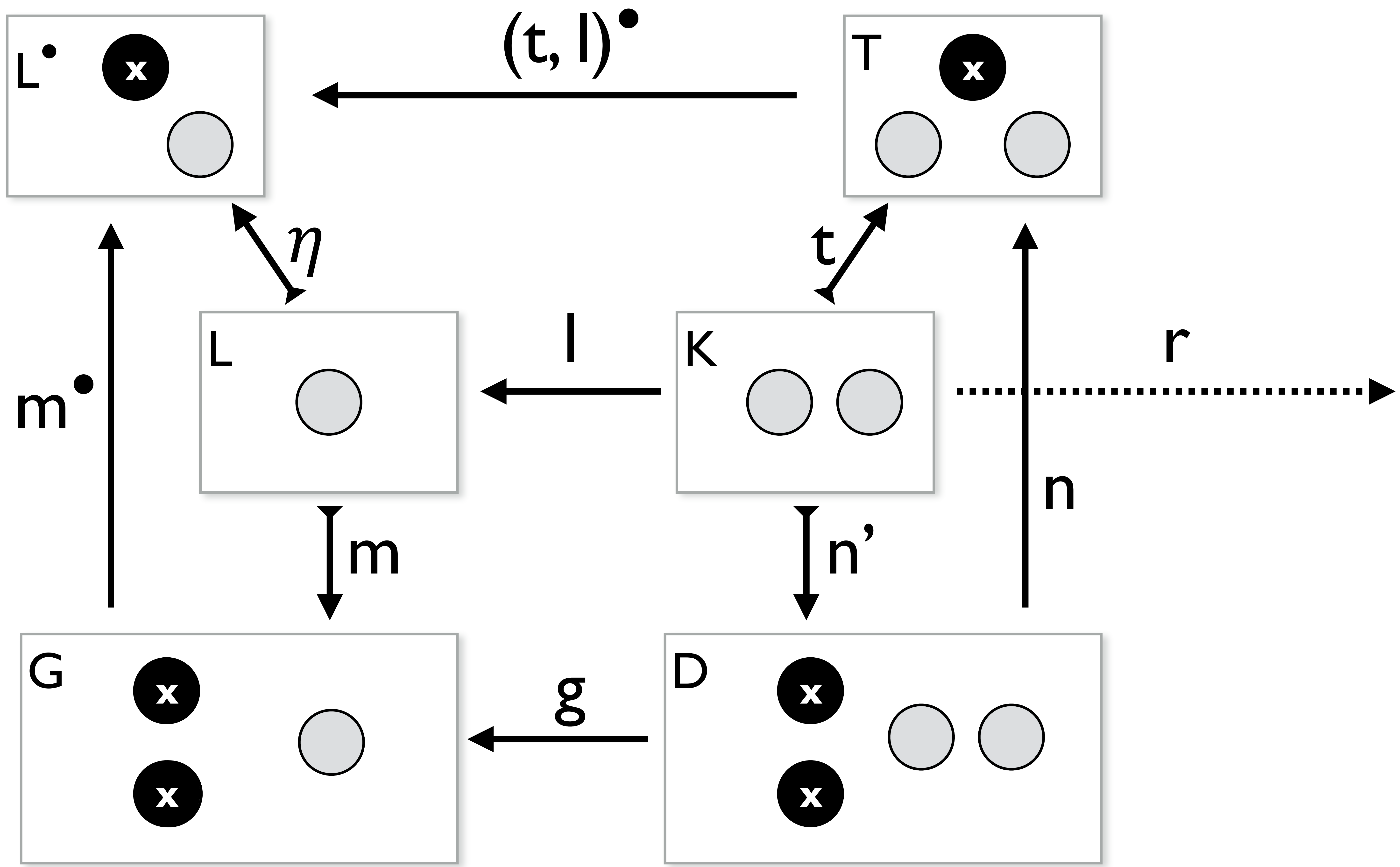
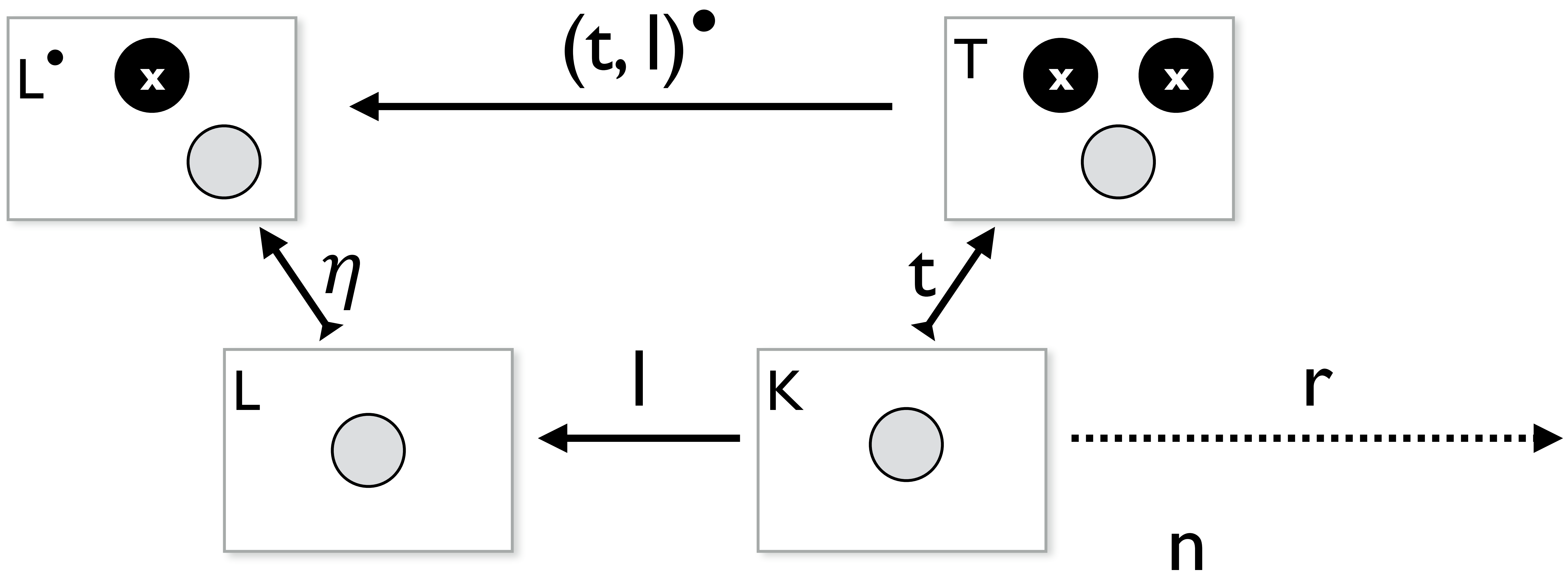


AGREE: Local Copies



AGREE: Global Copies



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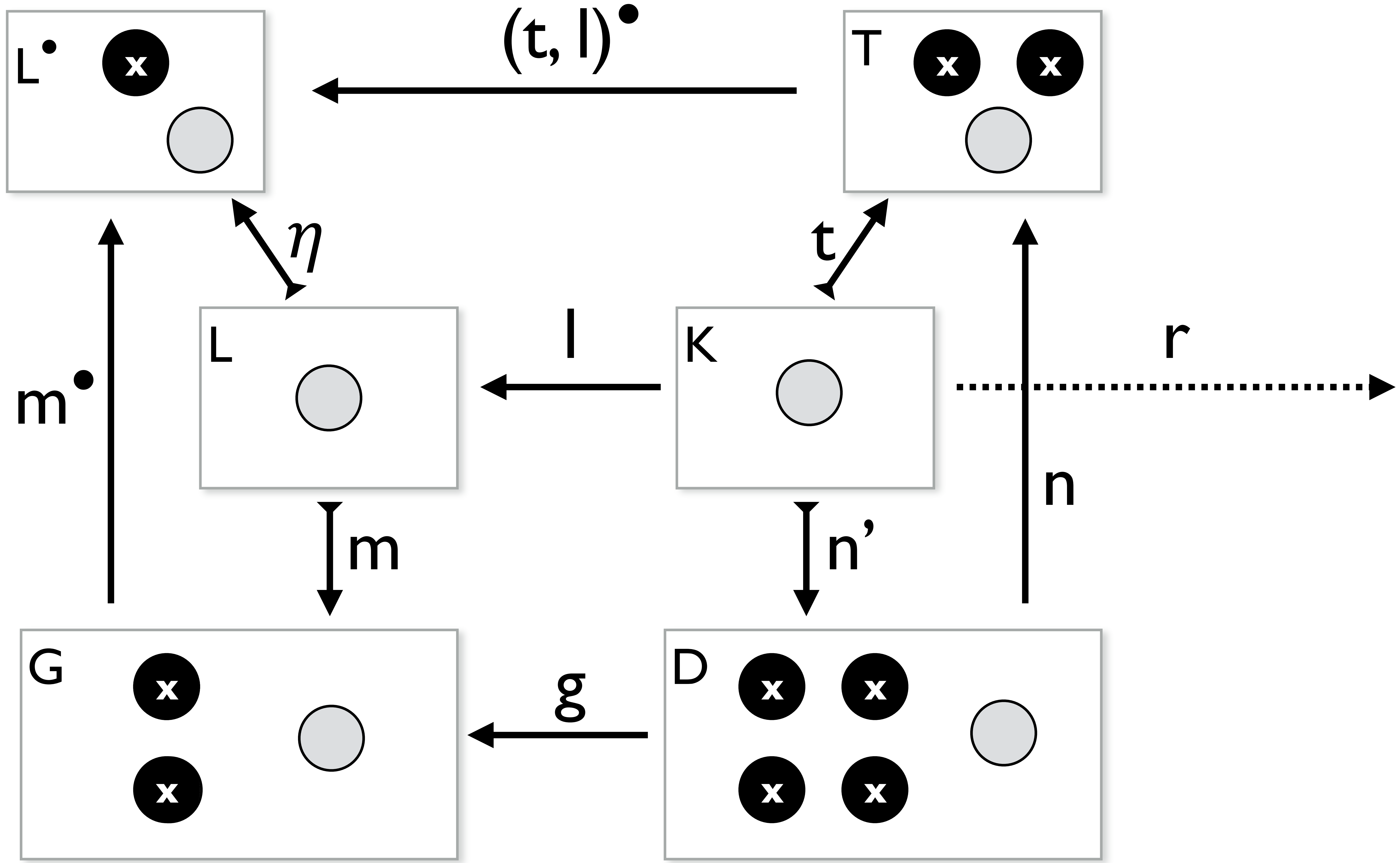
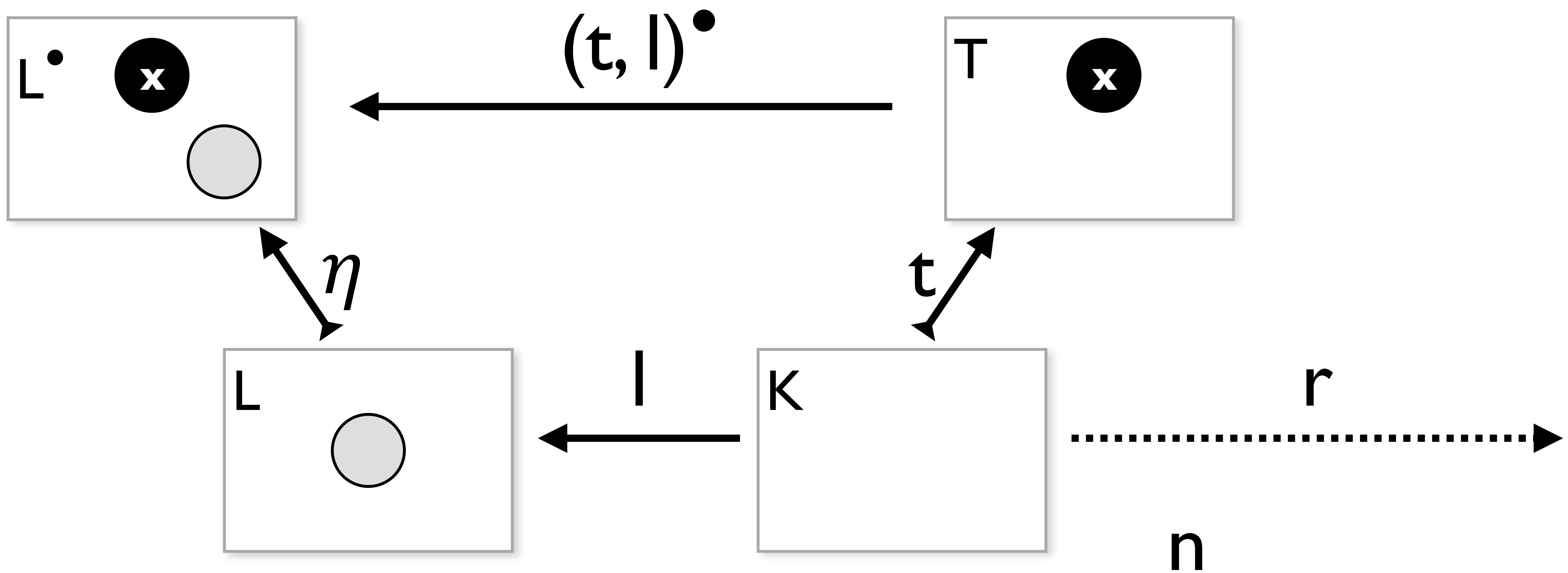


AGREE: Local Deletion



AGREE: Local Deletion

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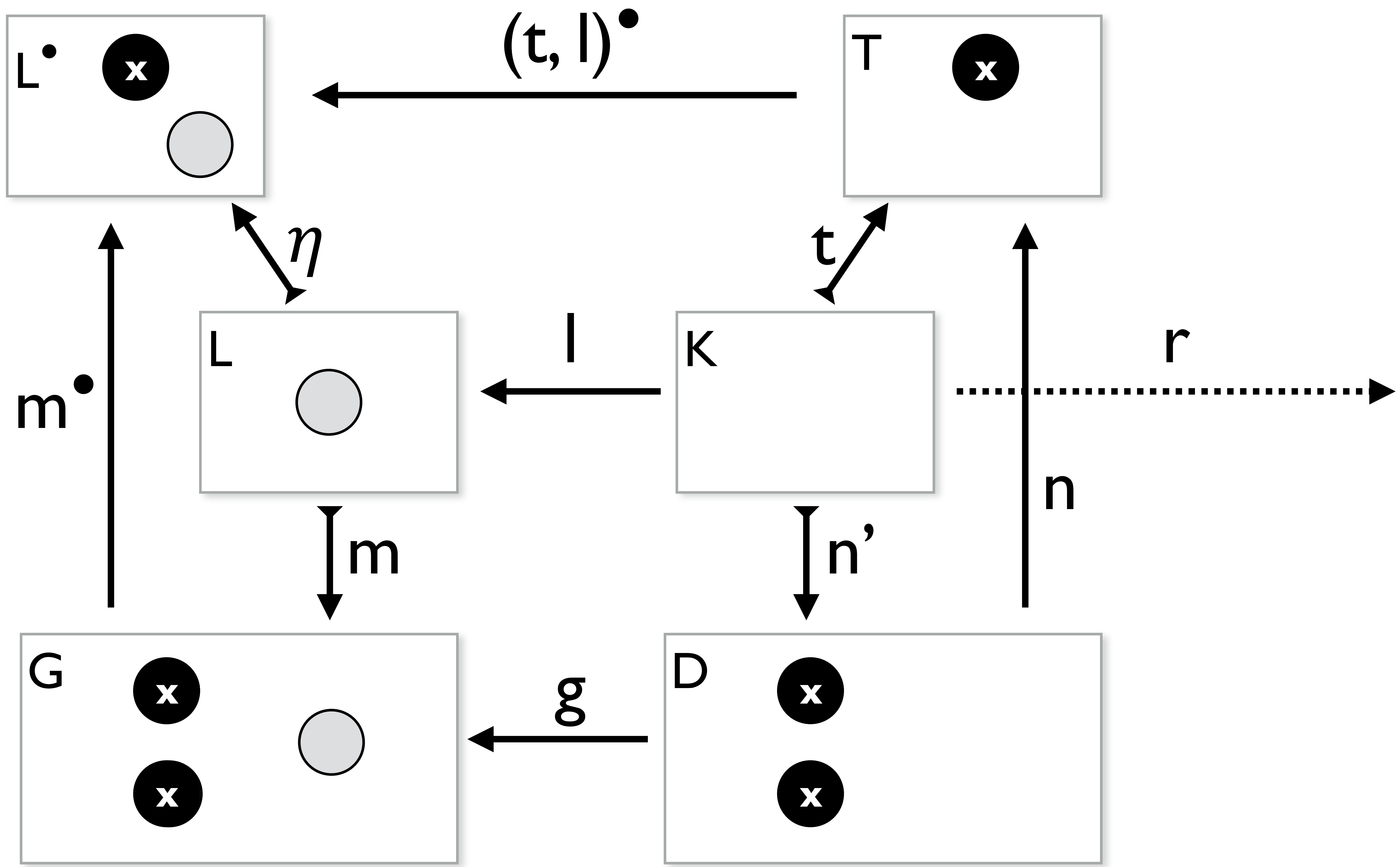

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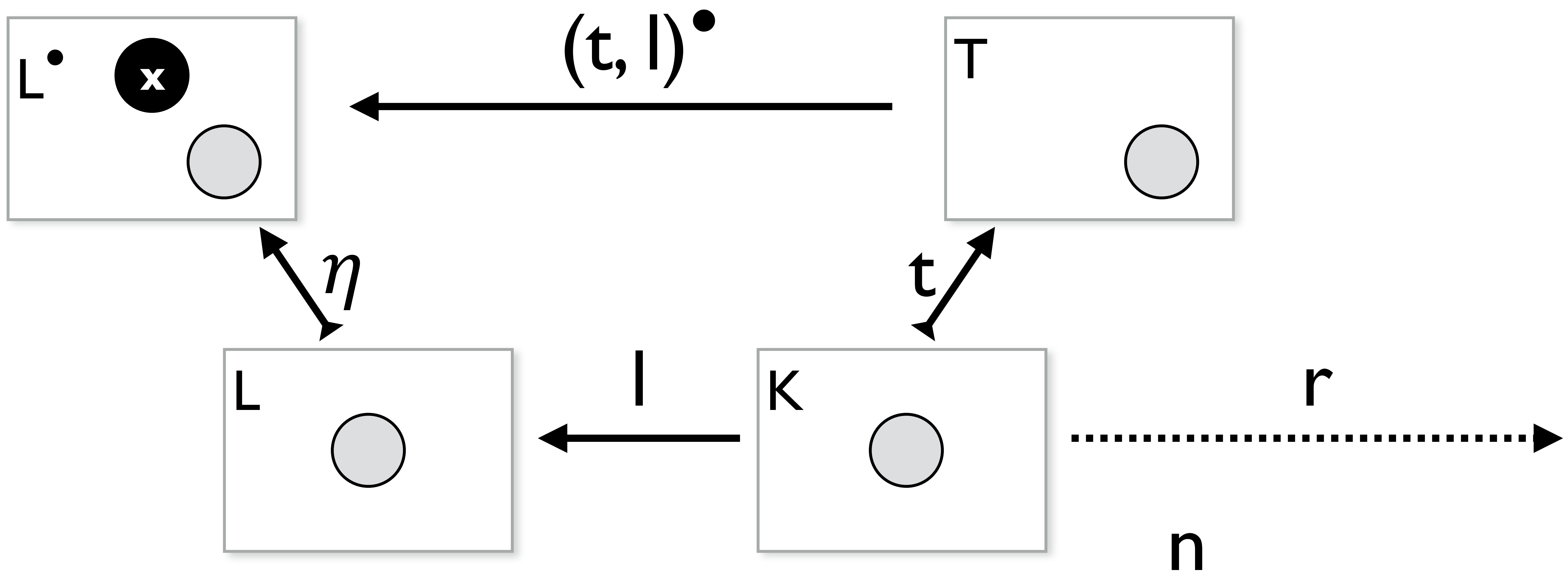

AGREE: Global Deletion

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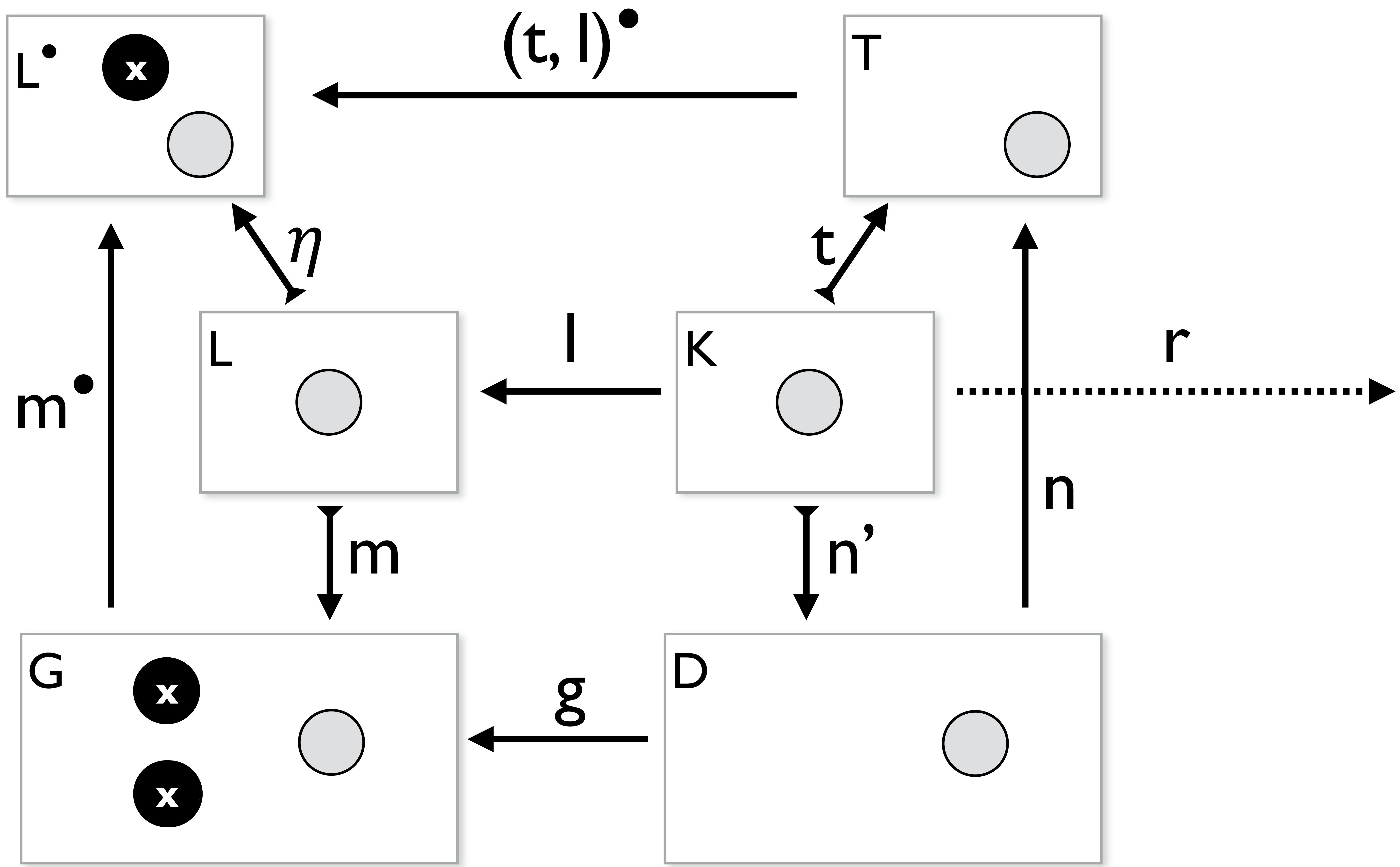
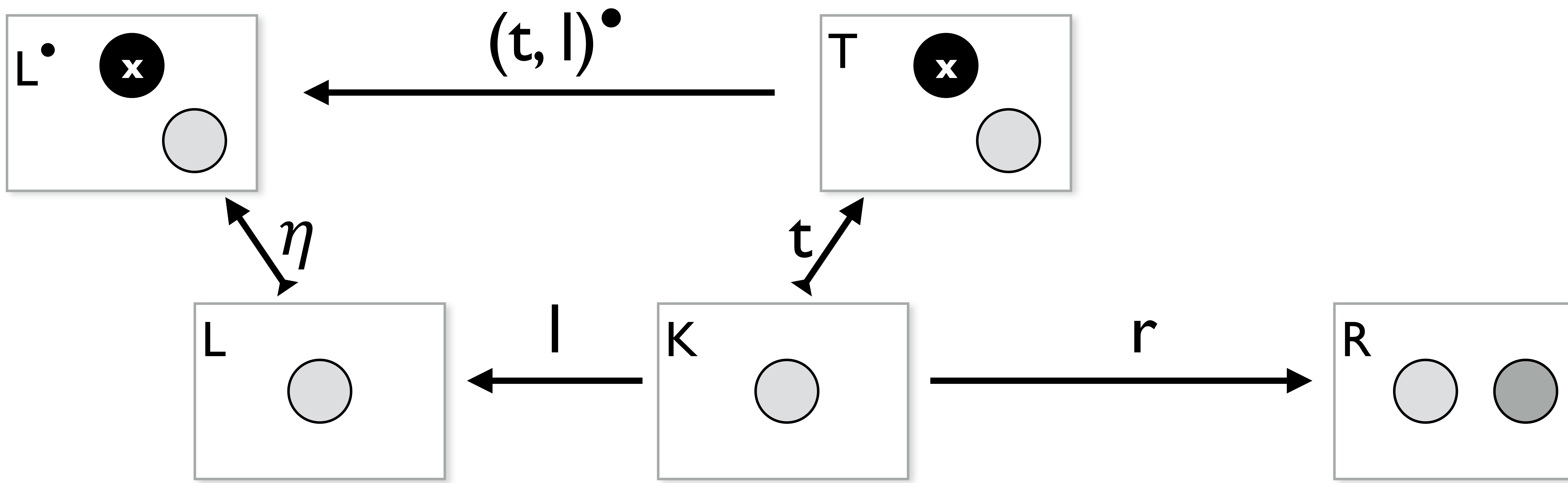
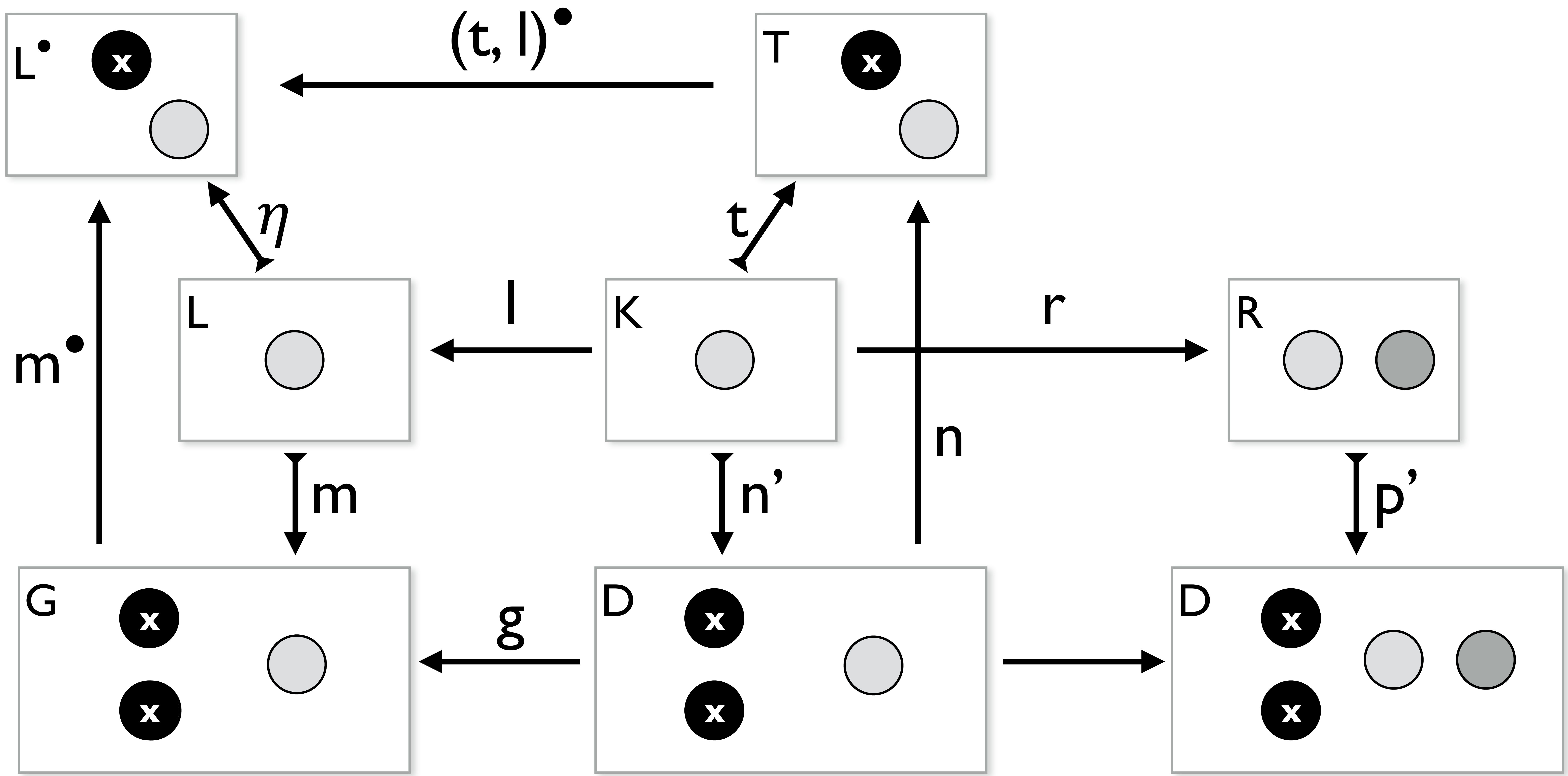


AGREE: Local Addition



AGREE: Local Addition



Gluing Construction

$$\begin{tikzcd} L & K \arrow[l, "l"'] \arrow[r, "r"] & R \\ X \arrow[u, "m"] \arrow[d, "n"] & & \\ G & & \end{tikzcd}$$

Gluing Construction

$$(r, l) \circ (q, p) = (n, m) \circ (h, g)$$

$$\begin{tikzcd}
L \arrow[d, phantom] & K \arrow[l, "l"'] \arrow[r, "r"] & R \\
X \arrow[u, "m"'] \arrow[d, "n"] \arrow[ur, phantom, "\textbf{(PB)}"] & Z \arrow[l, "x"'] \arrow[r, "y"] \arrow[u, "u"'] \arrow[d, "v"] \arrow[ur, phantom, "\textbf{(FPC)}"] \arrow[dl, phantom, "\textbf{(FPC)}"] \arrow[dr, phantom, "\textbf{(PO)}"] & Y \arrow[u, "p"'] \arrow[d, "q"] \\
G & D \arrow[l, "g"] \arrow[r, "h"'] & H
\end{tikzcd}$$

Gluing for DPO-Rewriting

$$\begin{tikzcd}
L & K \arrow[l, "l"'] \arrow[r, "r"] & R \\
X \arrow[u, "m"] \arrow[d, "n"'] \arrow[ur, phantom, "(PB)"] & Z \arrow[l, "x"'] \arrow[r, "y"] \arrow[u, "u"'] \arrow[d, "v"] \arrow[ur, phantom, "(FPC)"] & Y \arrow[u, "p"'] \arrow[d, "q"] \\
G \arrow[ur, phantom, "(FPC)"] & D \arrow[l, "g"] \arrow[r, "h"'] \arrow[ur, phantom, "(PO)"] & H
\end{tikzcd}$$

Gluing for DPO-Rewriting

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Monomorphism

Isomorphism

Pushout
Complement

$$
\begin{tikzcd}
L \arrow[dr, phantom, "\text{(PB)}"] & K \arrow[l, "l"'] \arrow[r, "r"] \arrow[dr, phantom, "\text{(FPC)}"] & R \\
X \arrow[u, "m"'] \arrow[d, "n"] \arrow[dr, phantom, "\text{(FPC)}"] & Z \arrow[l, "x"'] \arrow[r, "y"] \arrow[u, "u"'] \arrow[d, "v"] \arrow[dr, phantom, "\text{(PO)}"] & Y \arrow[u, "p"'] \arrow[d, "q"] \\
G & D \arrow[l, "g"] \arrow[r, "h"'] & H
\end{tikzcd}
$$

Gluing for SPO-Rewriting

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Monomorphism

Isomorphism

$$
\begin{tikzcd}
L & K \arrow[l, "l"'] \arrow[r, "r"] & R \\
X \arrow[u, "m"'] \arrow[ur, phantom, "(PB)"] & Z \arrow[l, "x"'] \arrow[r, "y"] \arrow[u, "u"'] \arrow[ur, phantom, "(FPC)"] \arrow[d, "v"] \arrow[dl, phantom, "(FPC)"] \arrow[dr, phantom, "(PO)"] & Y \arrow[u, "p"'] \\
G & D \arrow[l, "g"] \arrow[r, "h"'] & H
\arrow[from=2-1, to=3-1, "n"]
\arrow[from=2-3, to=3-3, "q"]
\end{tikzcd}
$$

Gluing for SqPO-Rewriting

$$
\begin{tikzcd}
L & K \arrow[l, "l"'] \arrow[r, "r"] \arrow[dl, phantom, "\text{(PB)}"] \arrow[dr, phantom, "\text{(FPC)}"] & R \\
X \arrow[u, "m"'] \arrow[d, "n"'] & Z \arrow[l, "x"'] \arrow[r, "y"] \arrow[u, "u"'] \arrow[d, "v"] \arrow[dl, phantom, "\text{(FPC)}"] \arrow[dr, phantom, "\text{(PO)}"] & Y \arrow[u, "p"'] \arrow[d, "q"] \\
G & D \arrow[l, "g"] \arrow[r, "h"'] & H
\end{tikzcd}
$$

Isomorphism

Gluing for AGREE-Rewriting

$$
\begin{tikzcd}
L & K \arrow[l, "l"'] \arrow[r, "r"] & R \\
X \arrow[u, "m"'] \arrow[d, "n"] & Z \arrow[l, "x"'] \arrow[r, "y"] \arrow[u, "u"'] \arrow[d, "v"] \arrow[ul, phantom, "\text{(PB)}"] \arrow[ur, phantom, "\text{(FPC)}"] \arrow[dl, phantom, "\text{(FPC)}"] \arrow[dr, phantom, "\text{(PO)}"] & Y \arrow[u, "p"'] \arrow[d, "q"] \\
G & D \arrow[l, "g"] \arrow[r, "h"'] & H
\end{tikzcd}
$$

Gluing for AGREE-Rewriting

Induced by base rule

Monomorphism

Induced by monic base match

Special monomorphism

$$
\begin{tikzcd}
L & K \arrow[l, "l"'] \arrow[r, "r"] & R \\
X \arrow[u, "m"] \arrow[d, "n"'] \arrow[ur, phantom, "\textbf{(PB)}"] \arrow[dr, phantom, "\textbf{(FPC)}"] & Z \arrow[l, "x"'] \arrow[r, "y"] \arrow[u, "u"'] \arrow[d, "v"] \arrow[ur, phantom, "\textbf{(FPC)}"] \arrow[dr, phantom, "\textbf{(PO)}"] & Y \arrow[u, "p"'] \arrow[d, "q"] \\
G & D \arrow[l, "g"] \arrow[r, "h"'] & H
\end{tikzcd}
$$

Gluing Construction

$$
\begin{tikzcd}
L & K \arrow[l, "l"'] \arrow[r, tail, "r"] & R \\
X \arrow[u, "m"'] \arrow[ur, phantom, "\text{(PB)}"] \arrow[d, tail, "n"] & Z \arrow[l, "x"'] \arrow[r, tail, "y"] \arrow[u, "u"'] \arrow[ur, phantom, "\text{(FPC)}"] \arrow[d, tail, "v"] & Y \arrow[u, "p"'] \arrow[d, tail, "q"] \\
G & D \arrow[l, "g"] \arrow[r, tail, "h"'] \arrow[ul, phantom, "\text{(FPC)}"] & H \arrow[ul, phantom, "\text{(PO)}"]
\end{tikzcd}
$$

Gluing Construction

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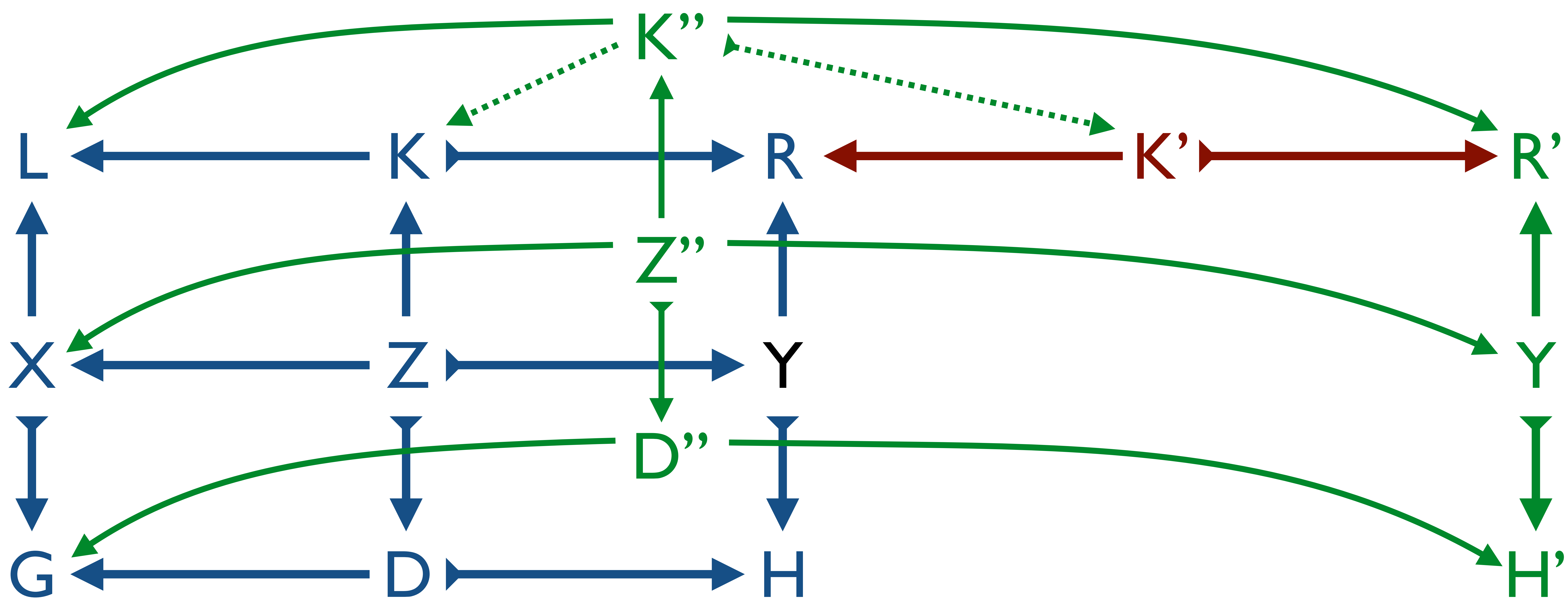
$$
\begin{tikzcd}
L & K \arrow[l, "l"'] \arrow[r, tail, "r"] & R \\
X \arrow[u, "m"] \arrow[ur, phantom, "\textbf{(PB)}"] & Z \arrow[l, "x"'] \arrow[r, tail, "y"] \arrow[u, "u"] \arrow[ur, phantom, "\textbf{(FPC)}"] \arrow[d, tail, "v"] \arrow[dr, phantom, "\textbf{(PO)}"] & Y \arrow[u, "p"'] \arrow[d, tail, "q"] \\
G & D \arrow[l, "g"] \arrow[r, tail, "h"'] \arrow[ul, phantom, "\textbf{(FPC)}"] & H \arrow[from=2-1, to=3-1, tail, "n"]
\end{tikzcd}
$$

Gluing Construction

$$
\begin{tikzcd}
L & K \arrow[l] \arrow[r, tail] & R \\
X \arrow[u] \arrow[d, tail] & Z \arrow[l] \arrow[r, tail] \arrow[u] \arrow[d, tail] & Y \arrow[u] \arrow[d, tail] \\
G & D \arrow[l] \arrow[r, tail] & H
\end{tikzcd}
$$

Gluing Construction



Gluing Construction

$$
\begin{tikzcd}
 & & K'' \arrow[dll, bend right] \arrow[dl, dotted] \arrow[drr, dotted] \arrow[drrr, bend left] & & & \\
L & K \arrow[l] \arrow[rr, tail] & & R & K' \arrow[l] \arrow[r, tail] & R' \\
 & & Z'' \arrow[uu] \arrow[dd, tail] \arrow[dll, bend right] \arrow[dl, dotted] \arrow[drr, dotted] \arrow[drrr, bend left] & & & \\
X \arrow[uu] \arrow[dd, tail] & Z \arrow[uu] \arrow[dd, tail] \arrow[l] \arrow[rr, tail] & & Y \arrow[uu] \arrow[dd, tail] & Z' \arrow[uu, dotted] \arrow[dd, dotted, tail] \arrow[l, dotted] \arrow[r, dotted, tail] & Y \arrow[uu] \arrow[dd, tail] \\
 & & D'' \arrow[dll, bend right] \arrow[dl, dotted] \arrow[drr, dotted] \arrow[drrr, bend left] & & & \\
G & D \arrow[l] \arrow[rr, tail] & & H & D' \arrow[l] \arrow[r, tail] & H'
\end{tikzcd}
$$

Gluing Construction

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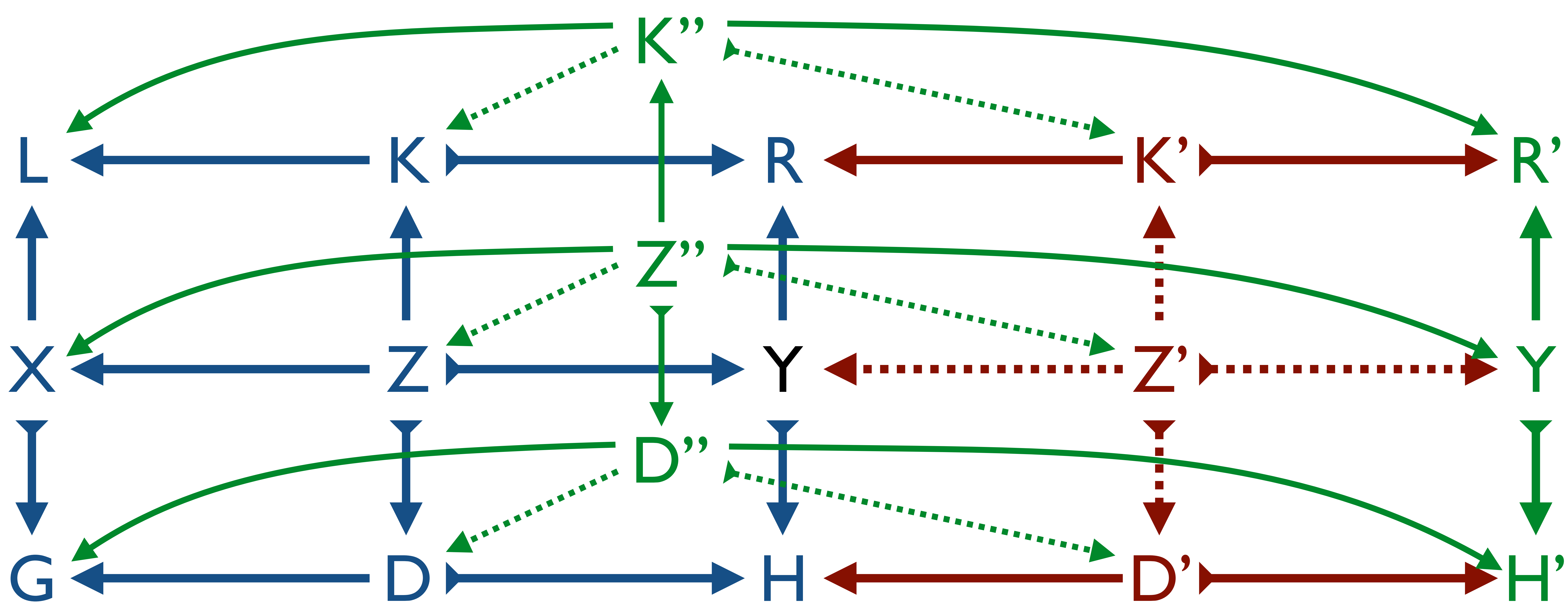
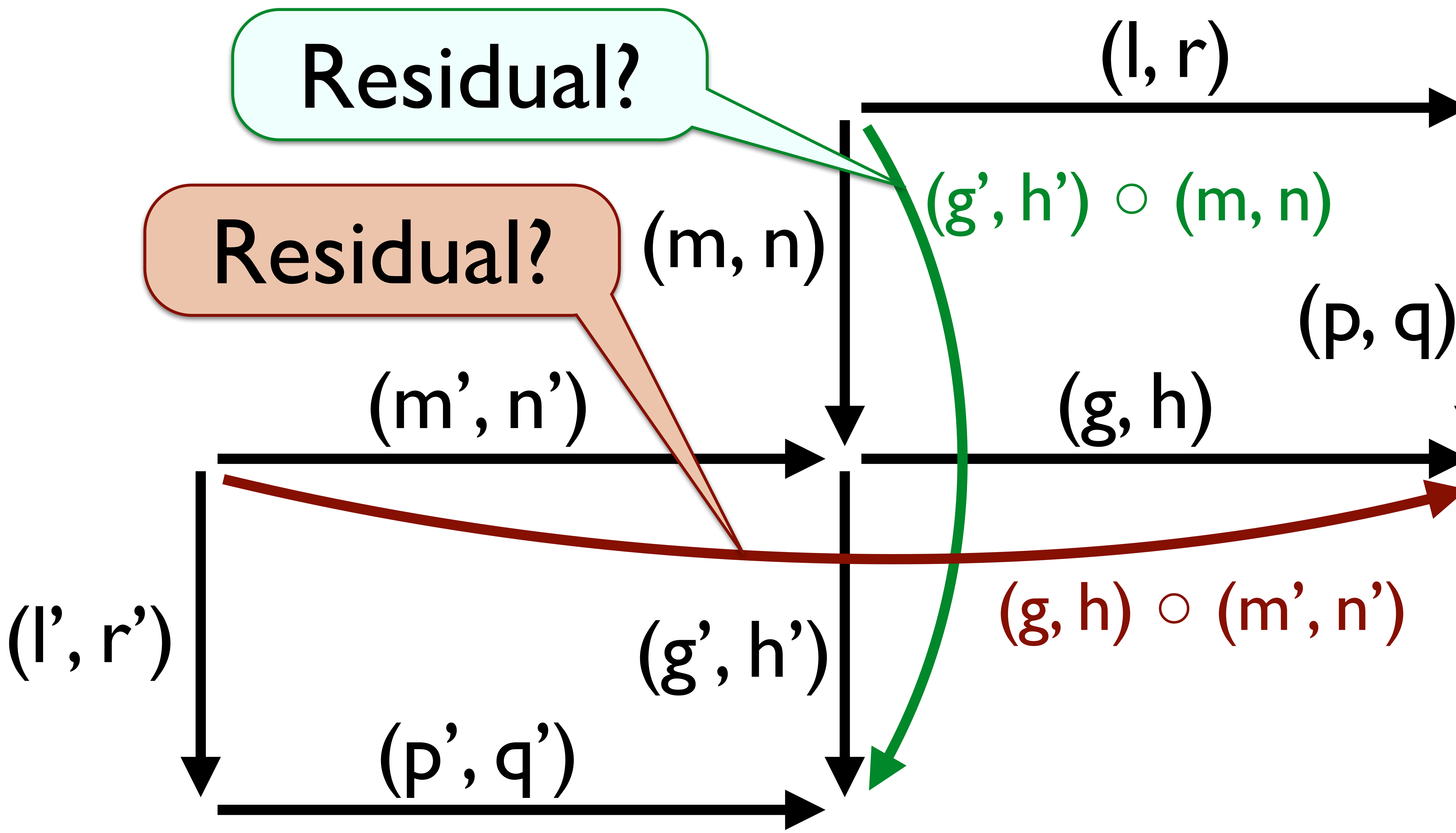


Gluing diagrams compose and decompose like pushouts

Parallel Independence

$$
\begin{tikzcd}
 & \cdot \arrow[r, "{(l, r)}"] \arrow[d, "{(m, n)}"'] & \cdot \arrow[d, "{(p, q)}"] \\
\cdot \arrow[r, "{(m', n')}"] \arrow[d, "{(l', r')}"'] & \cdot \arrow[r, "{(g, h)}"] \arrow[d, "{(g', h')}"'] & \cdot \\
\cdot \arrow[r, "{(p', q')}"] & \cdot &
\end{tikzcd}
$$

Parallel Independence



Parallel Independence

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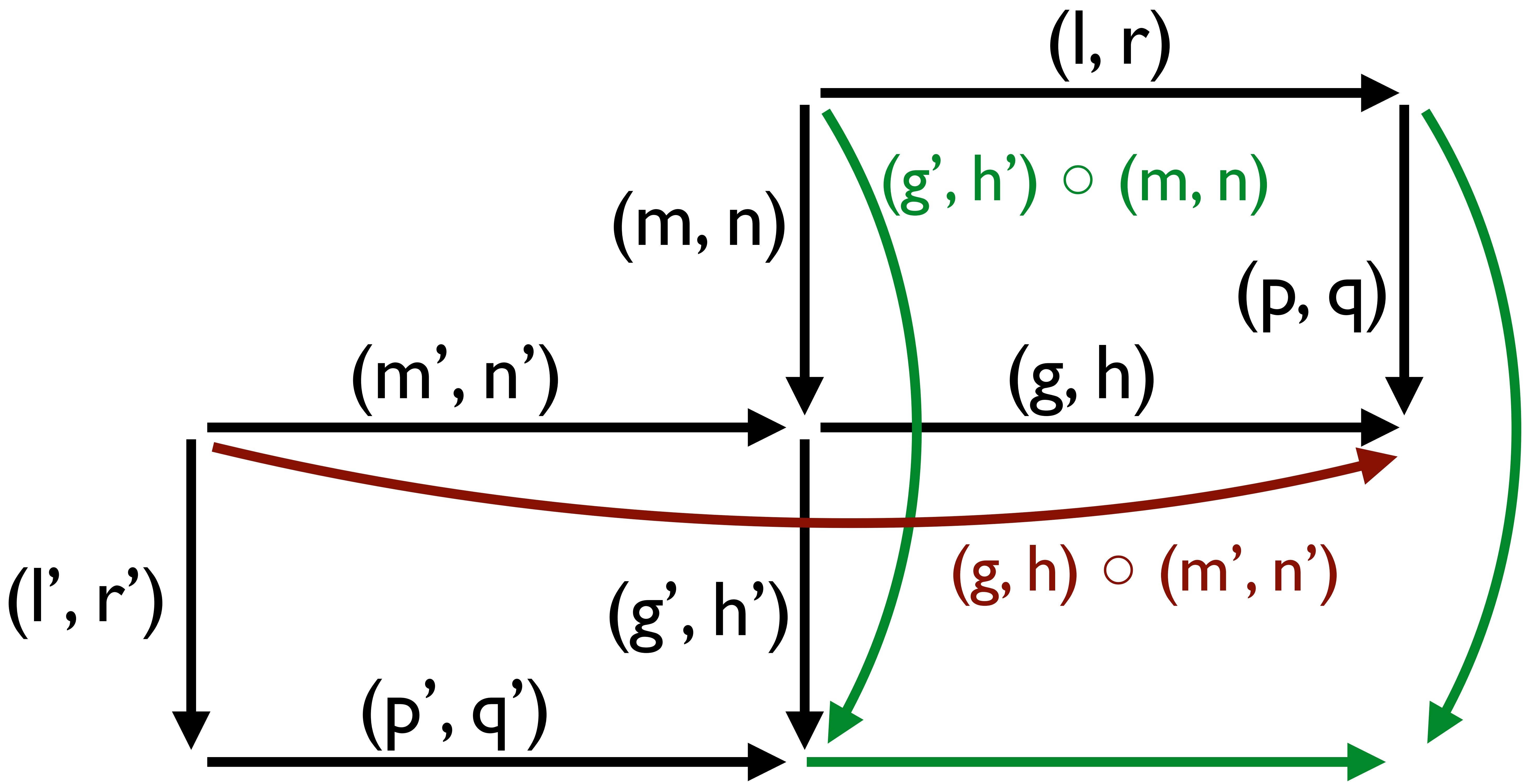
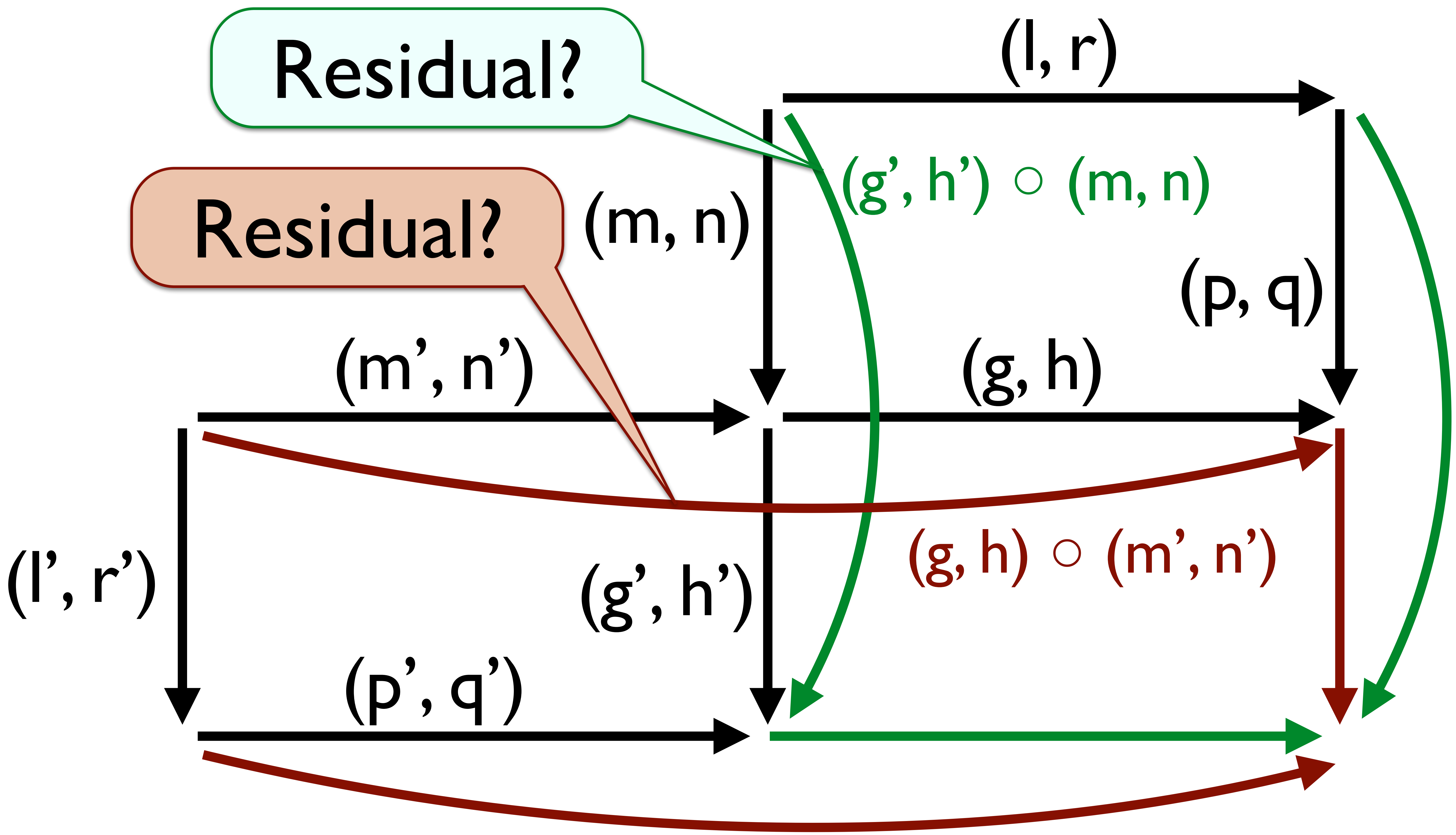
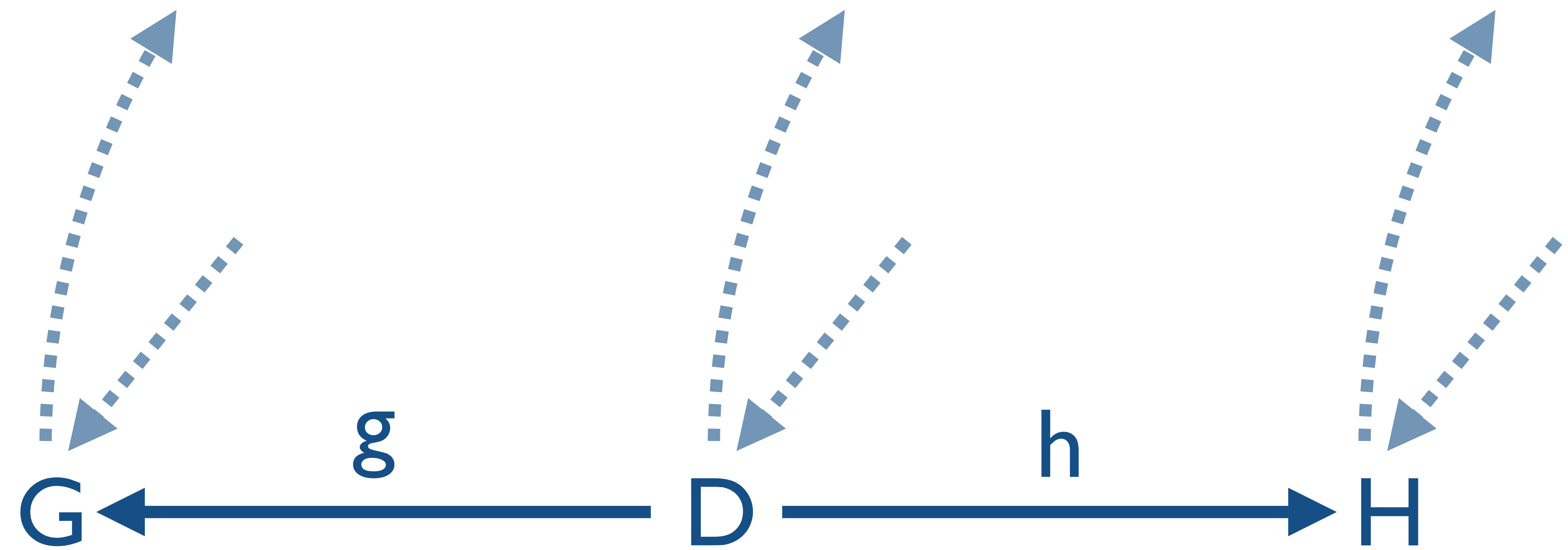


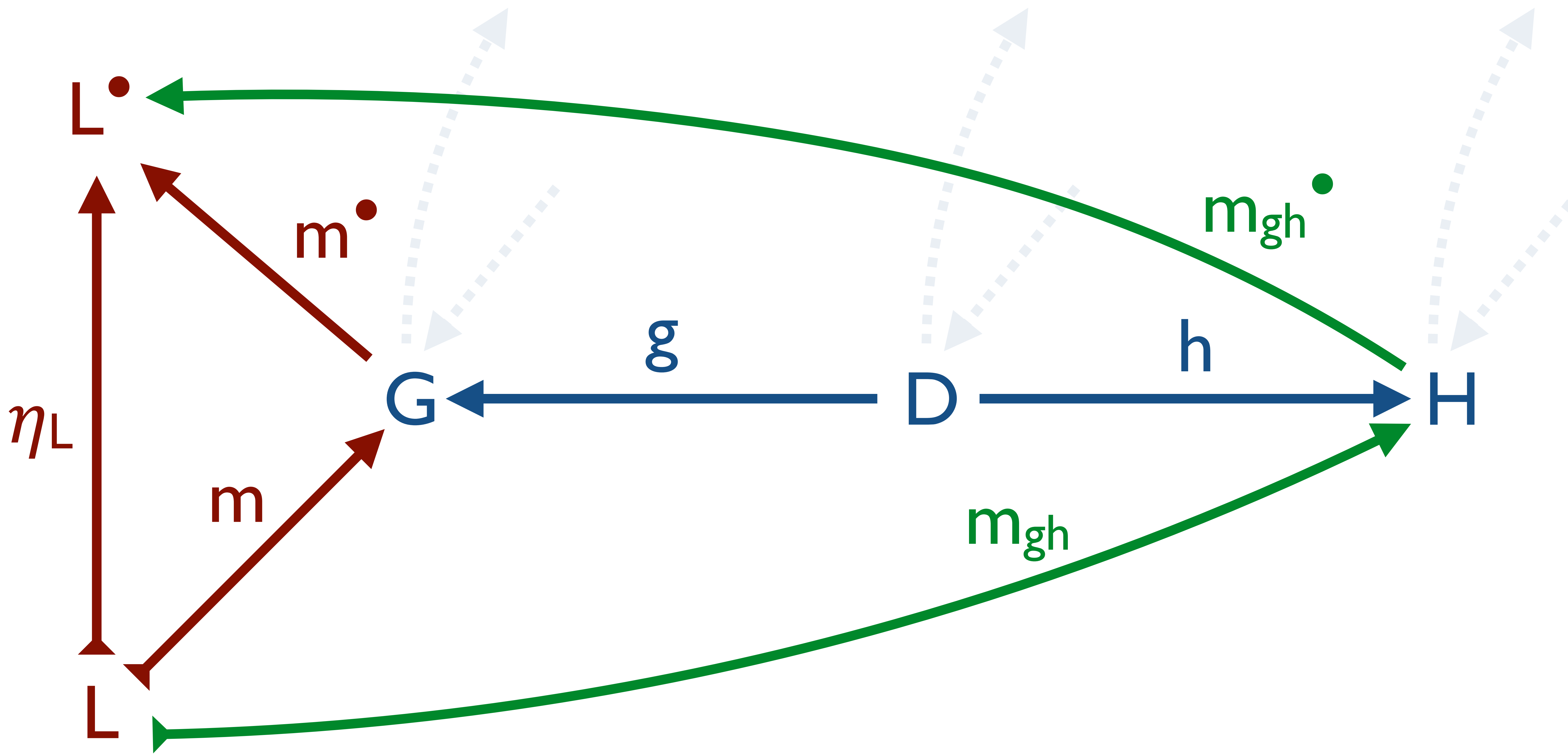
Parallel Independence



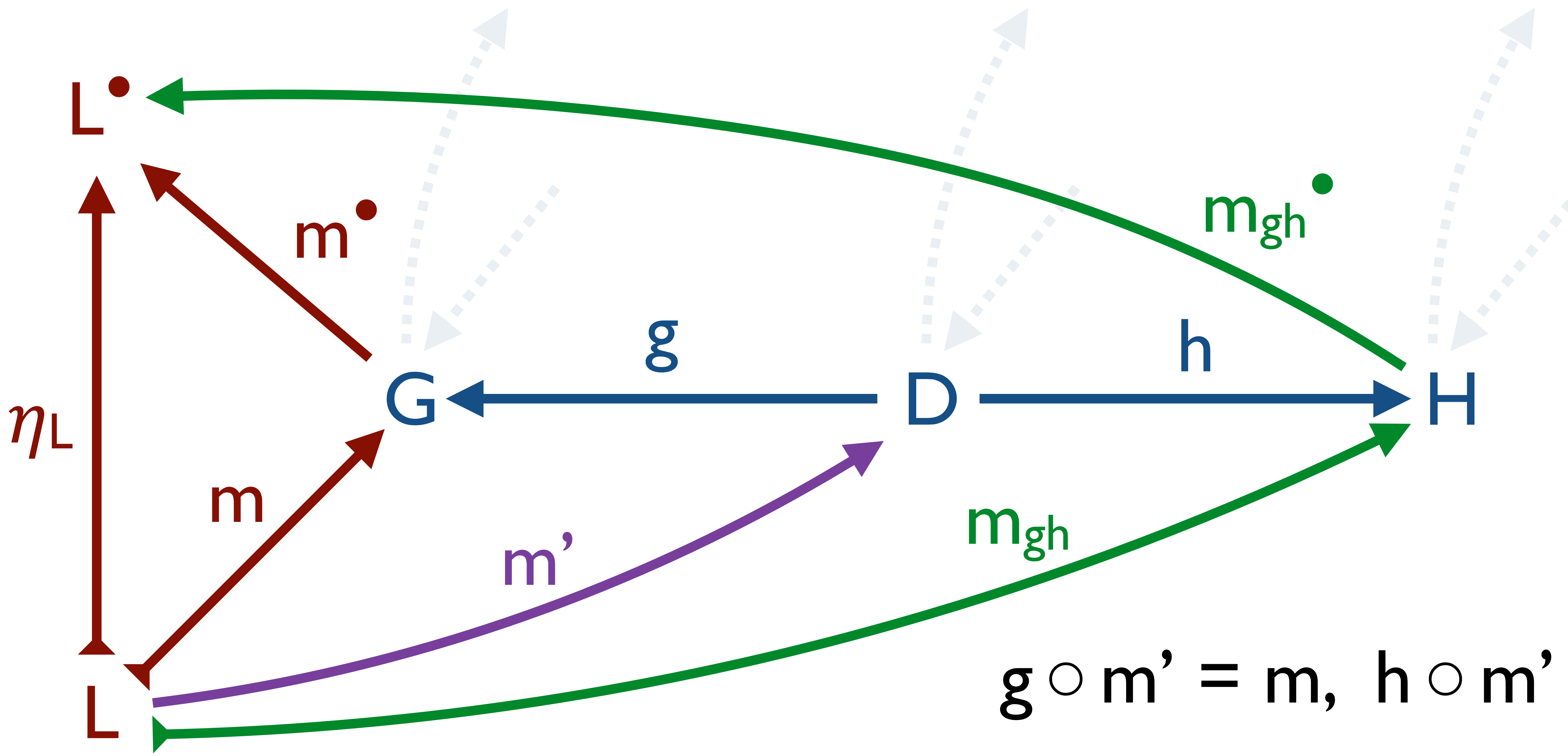
Residual



Residual



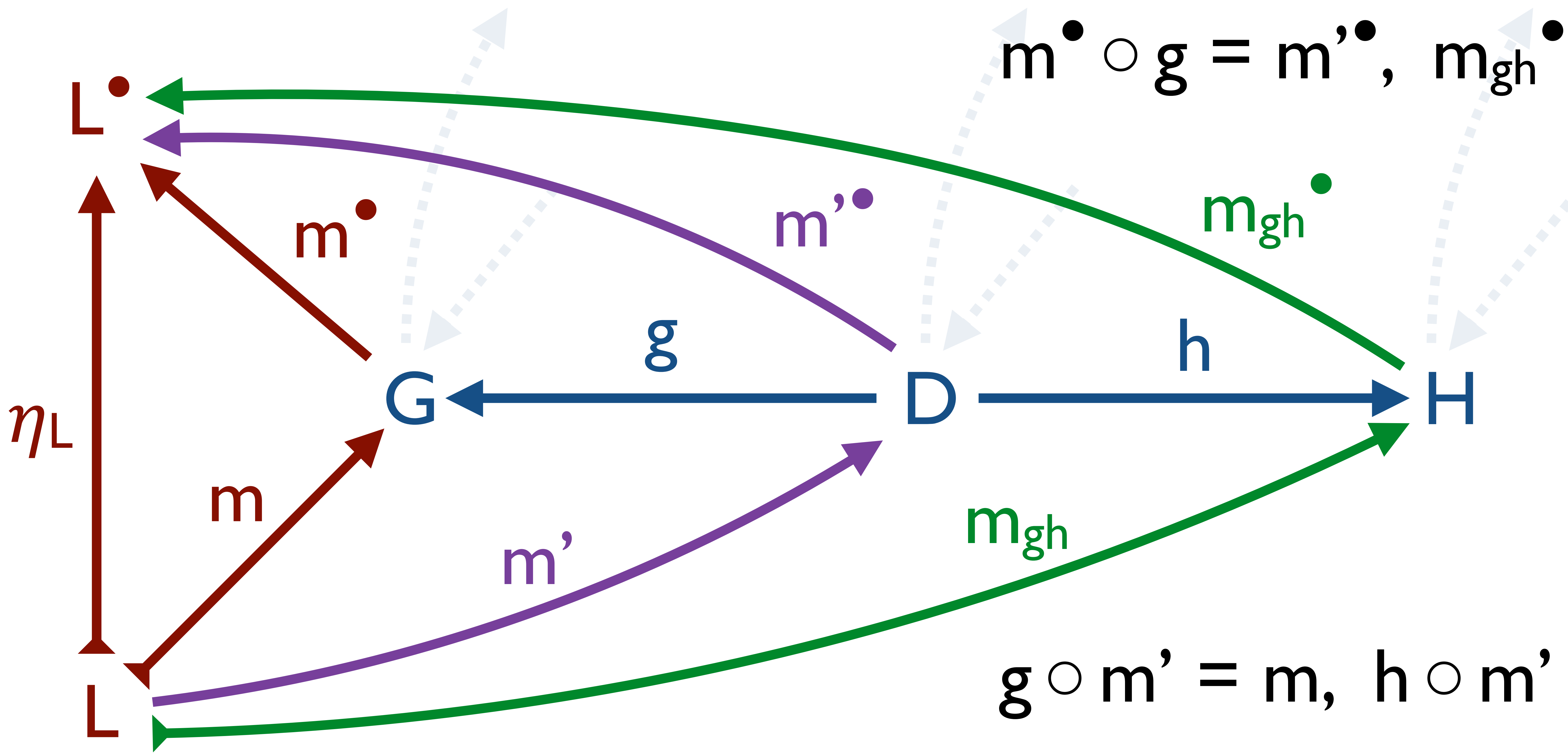
Residual



$g \circ m' = m, \;\; h \circ m' = m_{gh}$

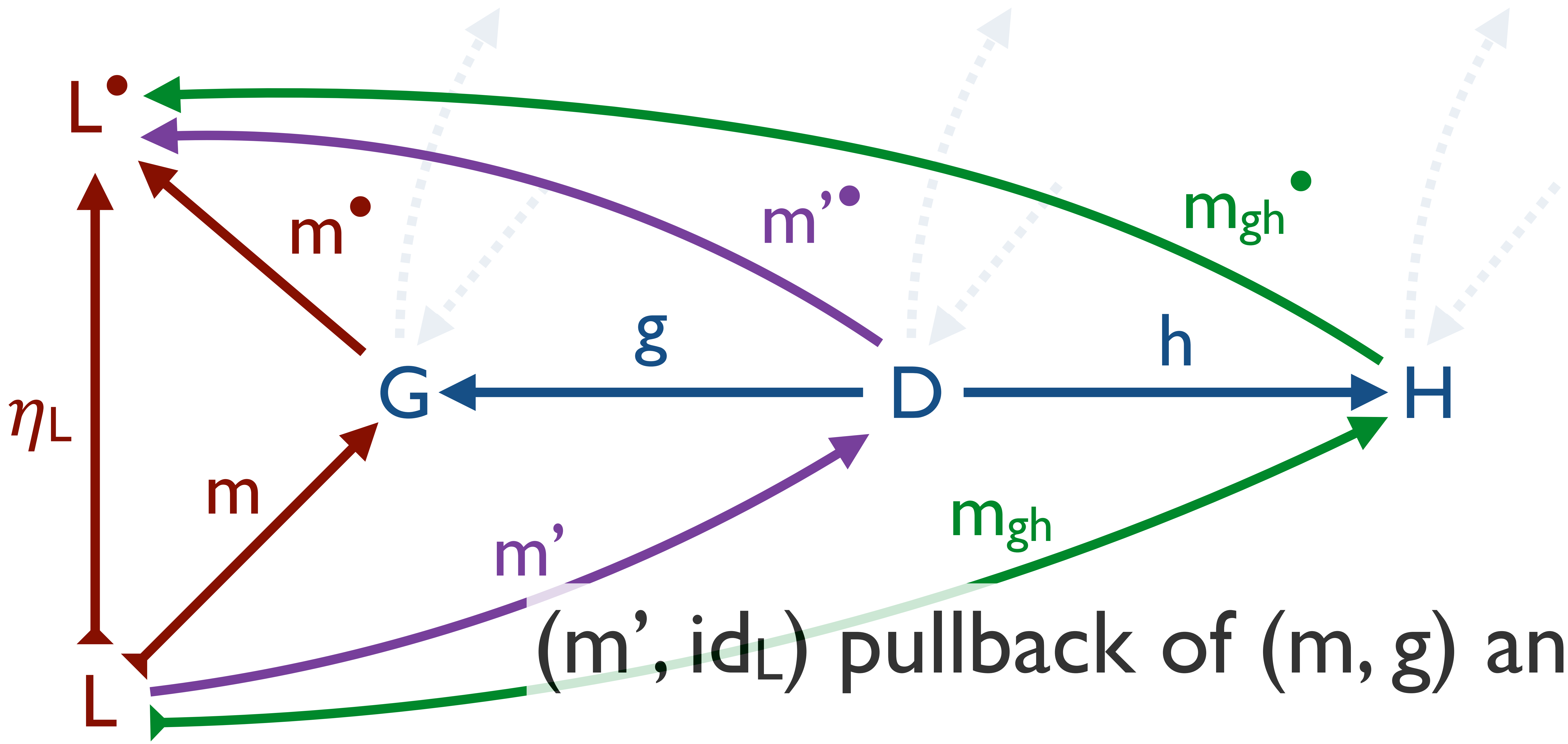
Residual

$m^\bullet \circ g = m'^\bullet, \; m_{gh}^\bullet \circ h = m'^\bullet$



$g \circ m' = m, \; h \circ m' = m_{gh}$

Residual



(m', id_L) pullback of (m, g) and (h, m_{gh})

Residual

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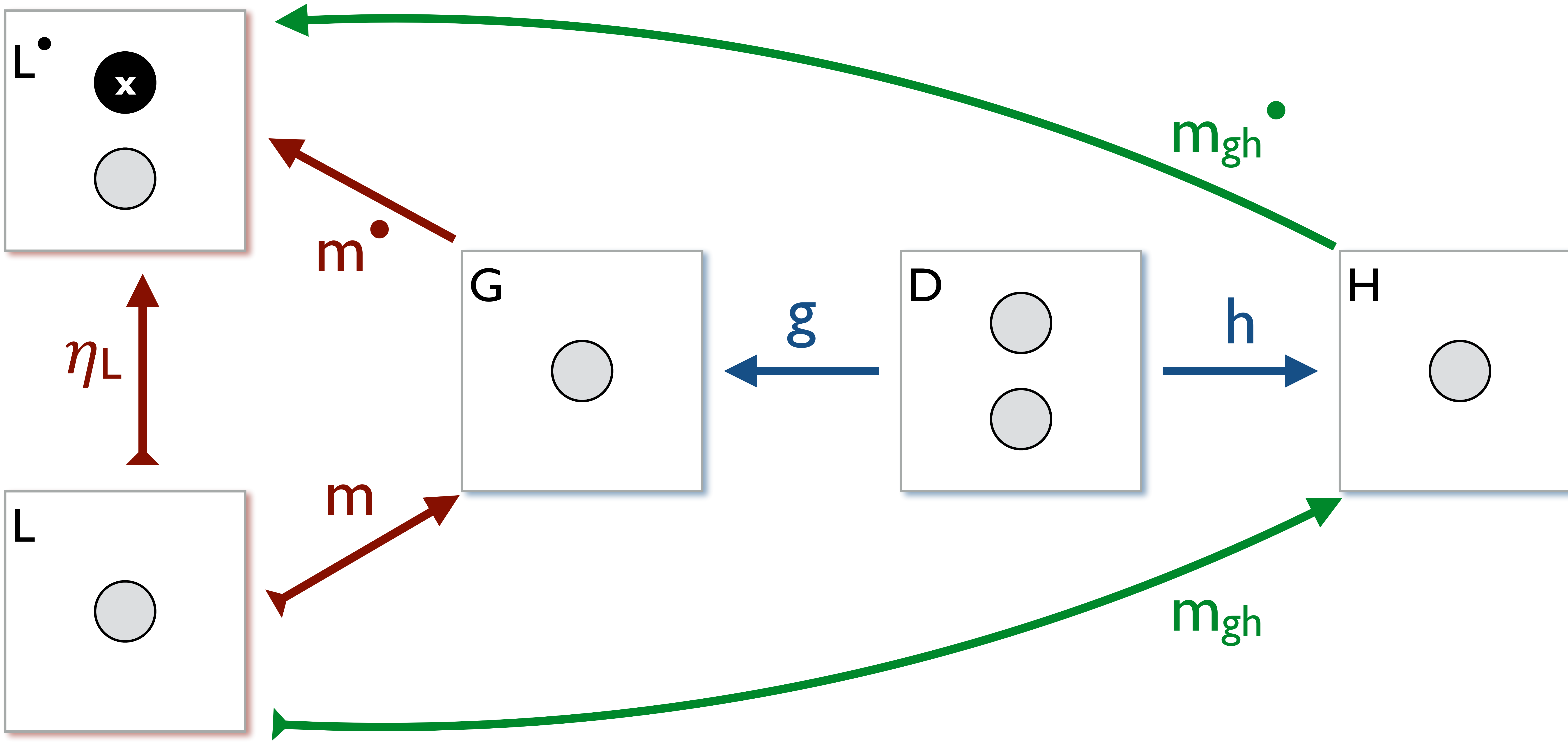
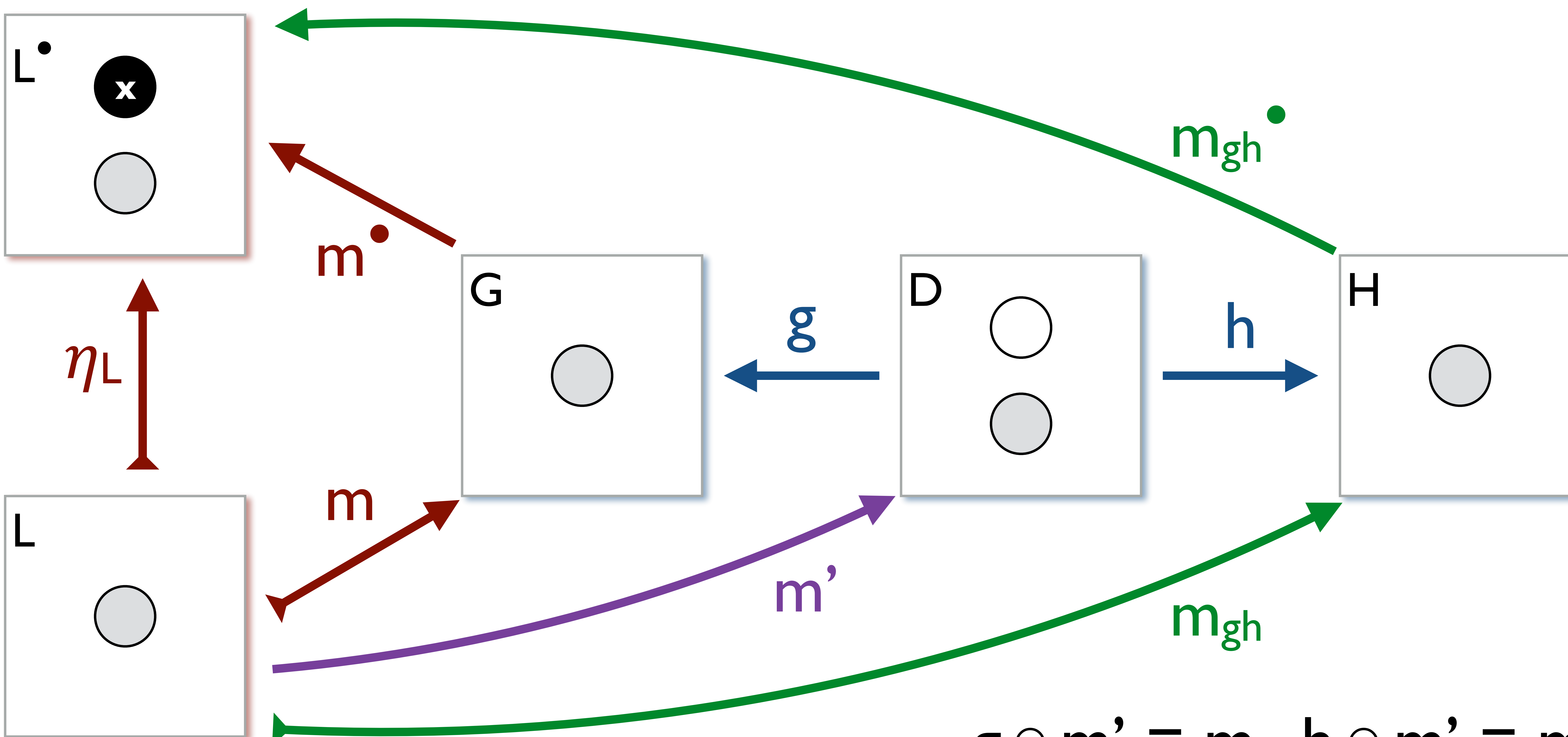


Residual



$g \circ m' = m, \; h \circ m' = m_{gh}$

Residual

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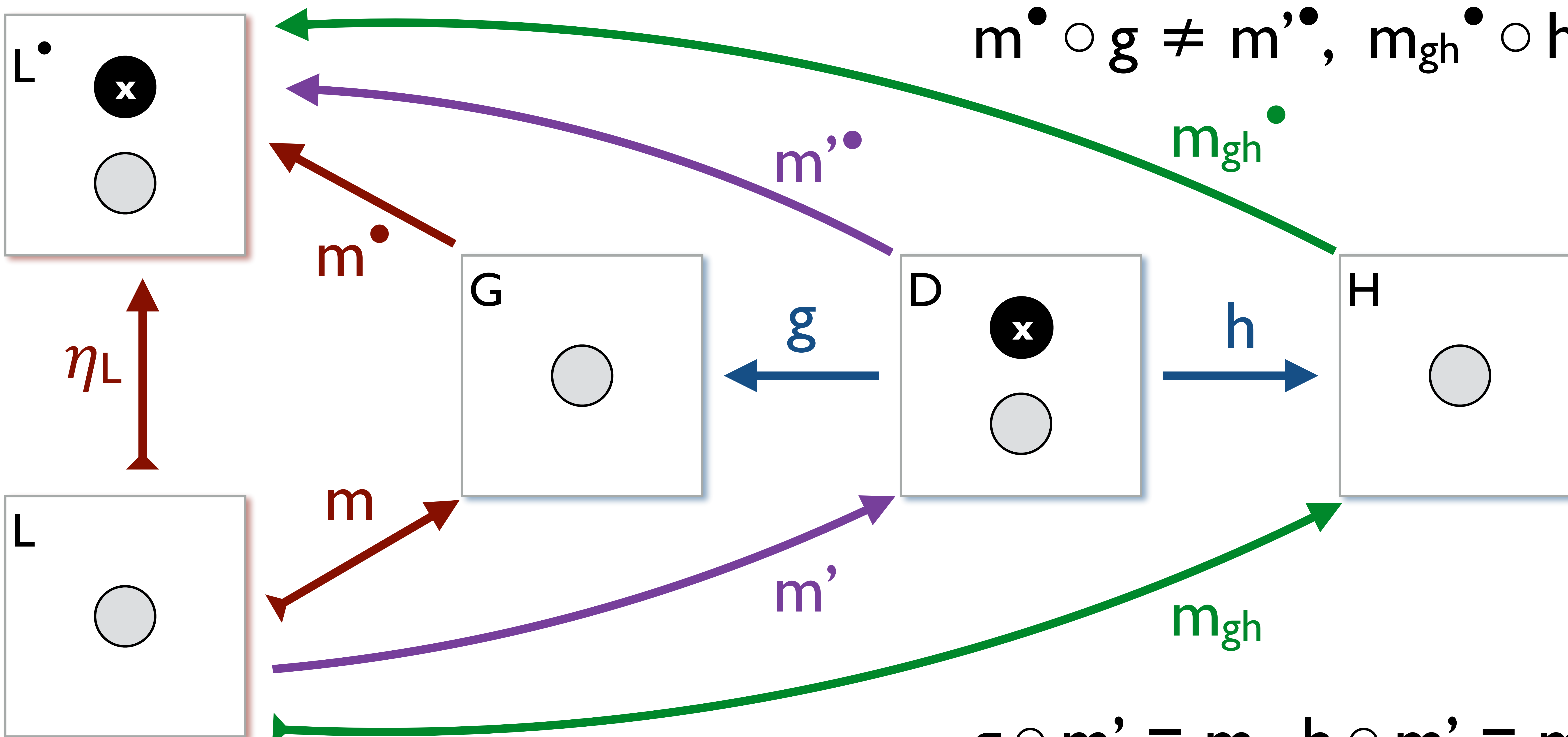

$$m^{\bullet} \circ g \neq m'^{\bullet}, \;\; m_{gh}{}^{\bullet} \circ h \neq m'^{\bullet}$$

$$g \circ m' = m, \;\; h \circ m' = m_{gh}$$

Residual

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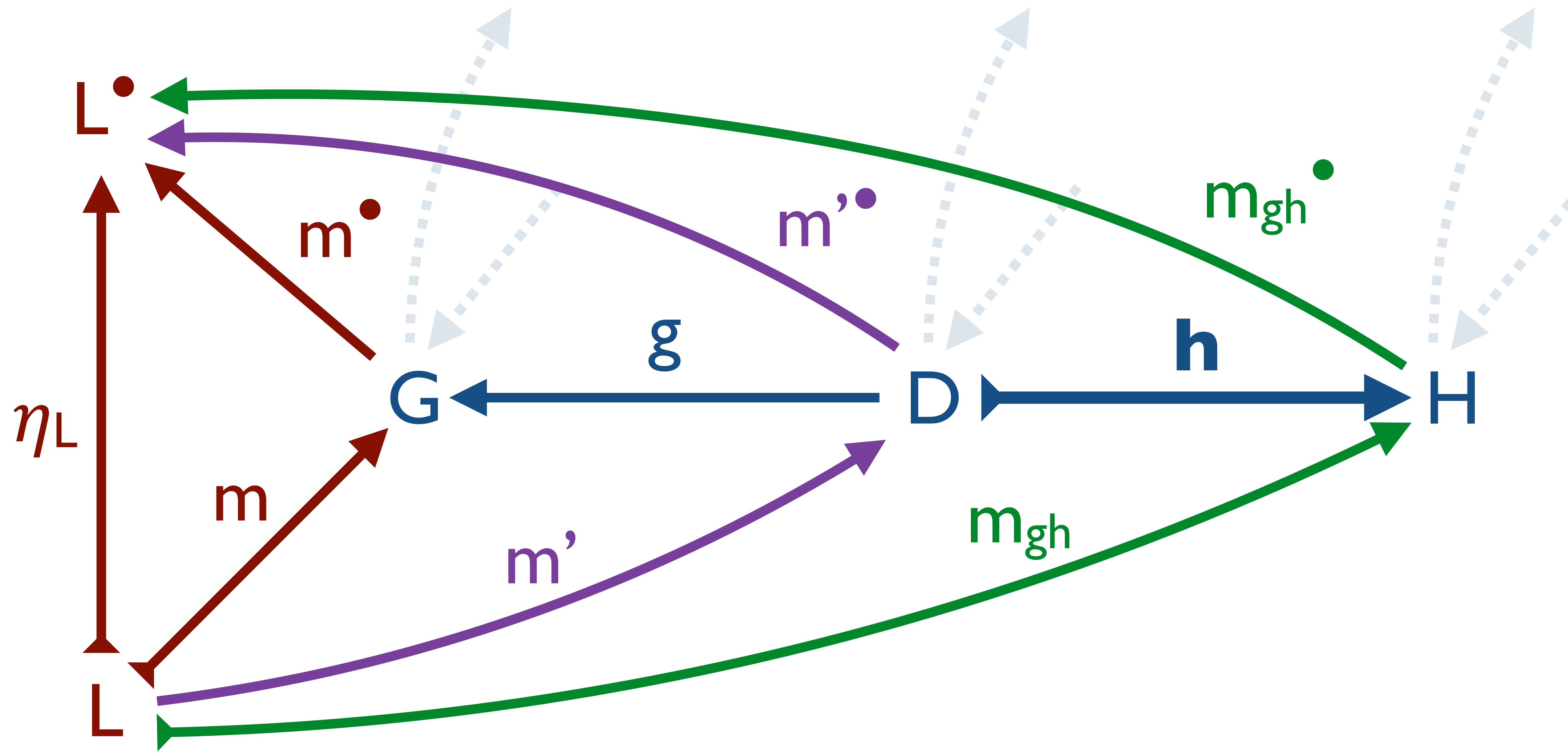

(m', id_L) pullback of (m, g) and (h, m_{gh})

Residual

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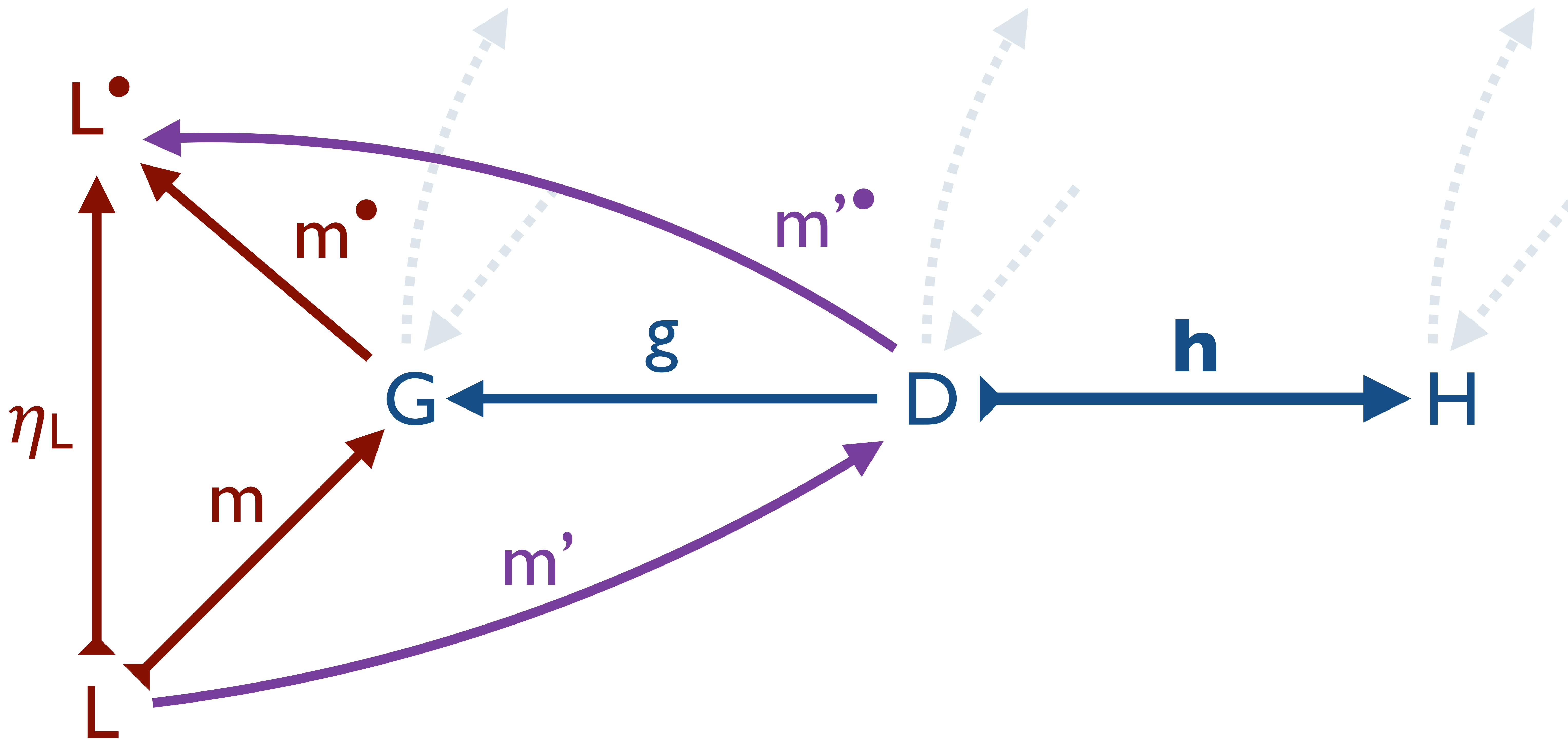
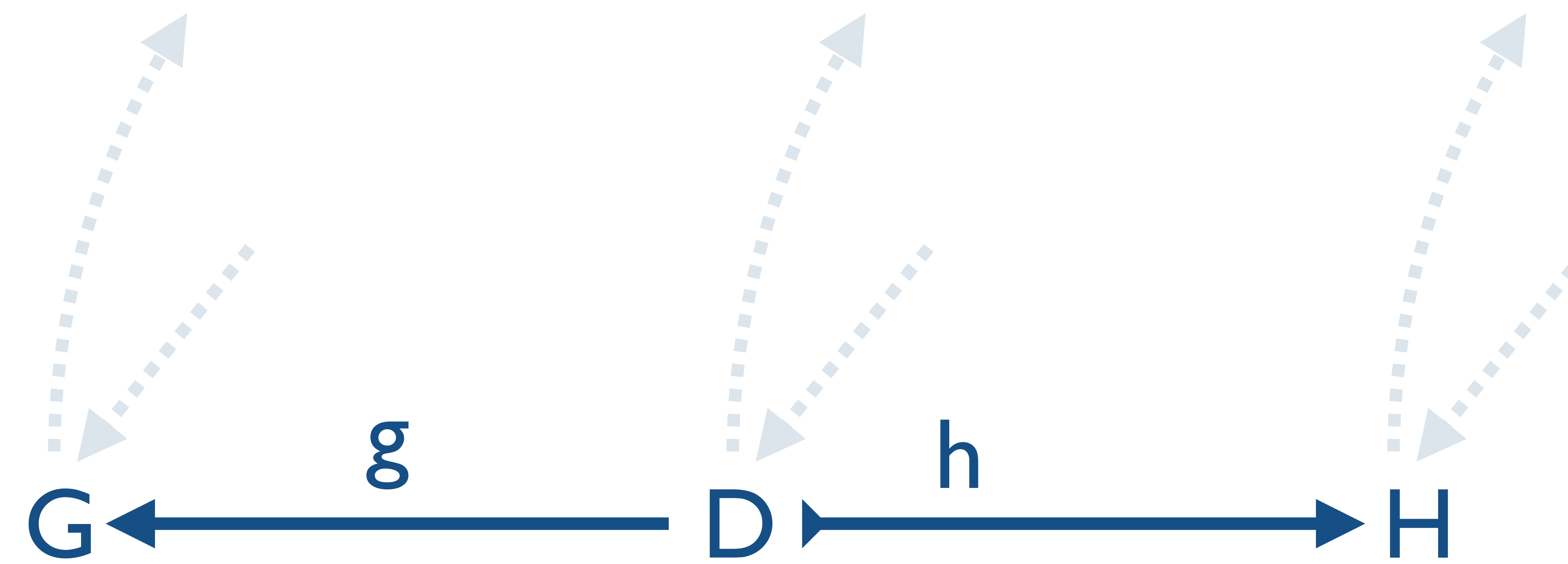


(m', id_L) pullback of (m, g)

Residual



Residual

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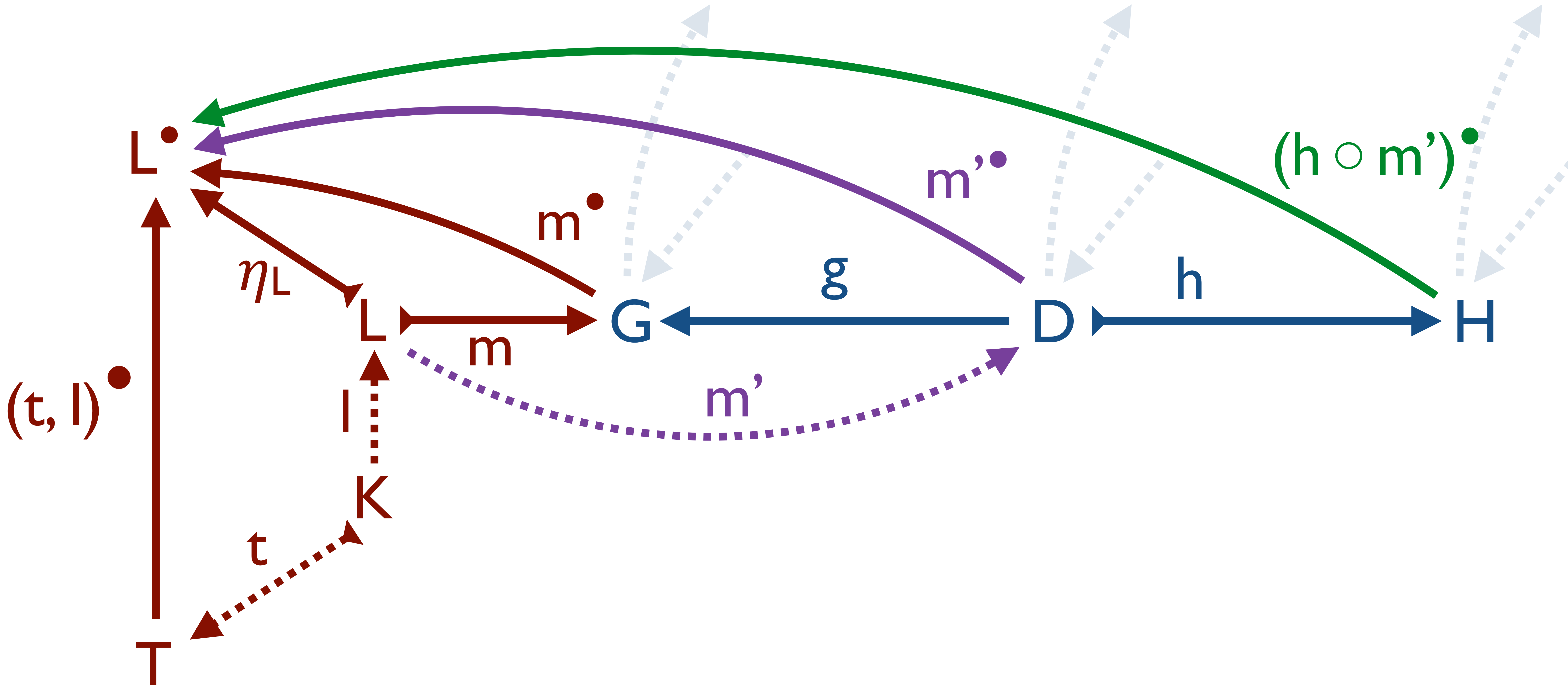
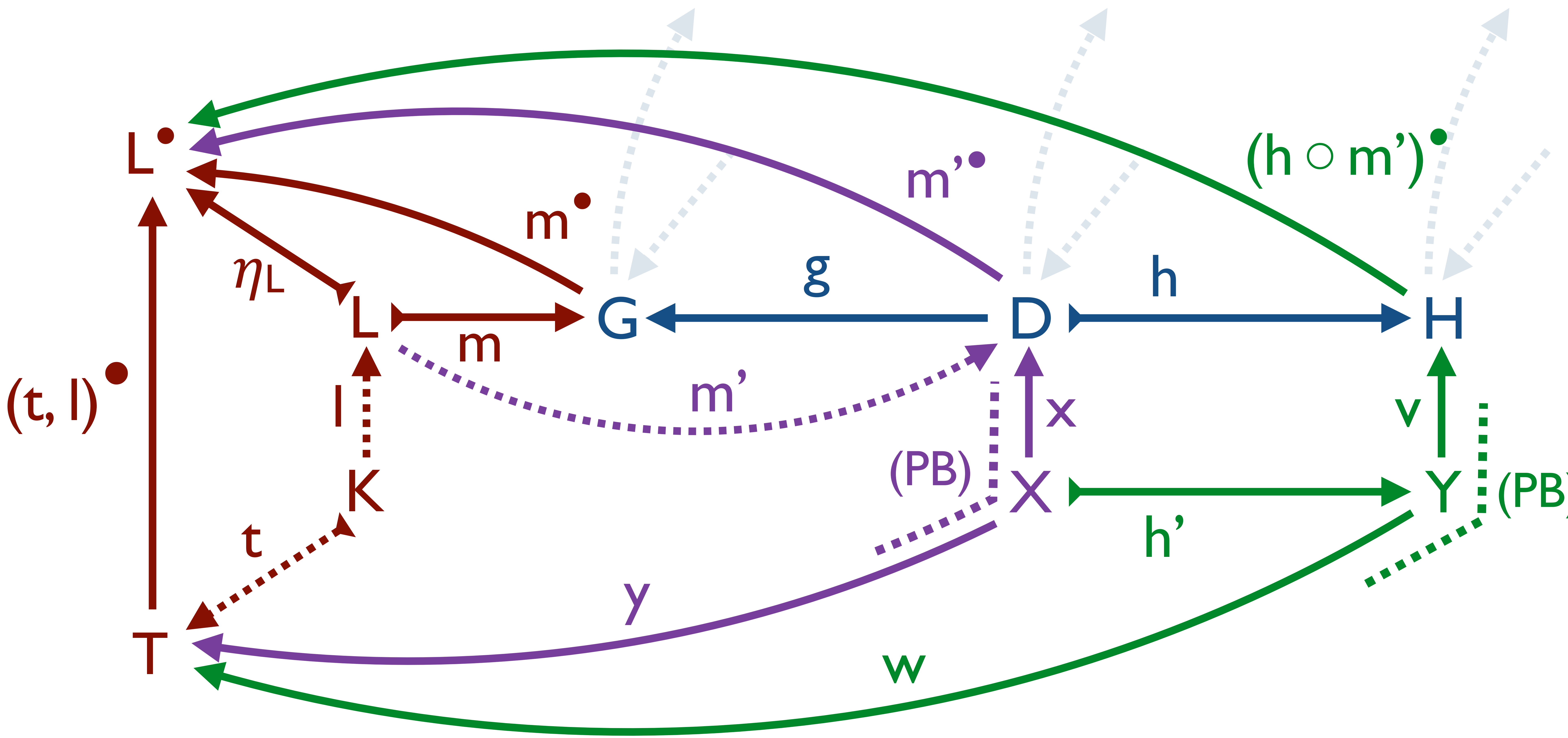
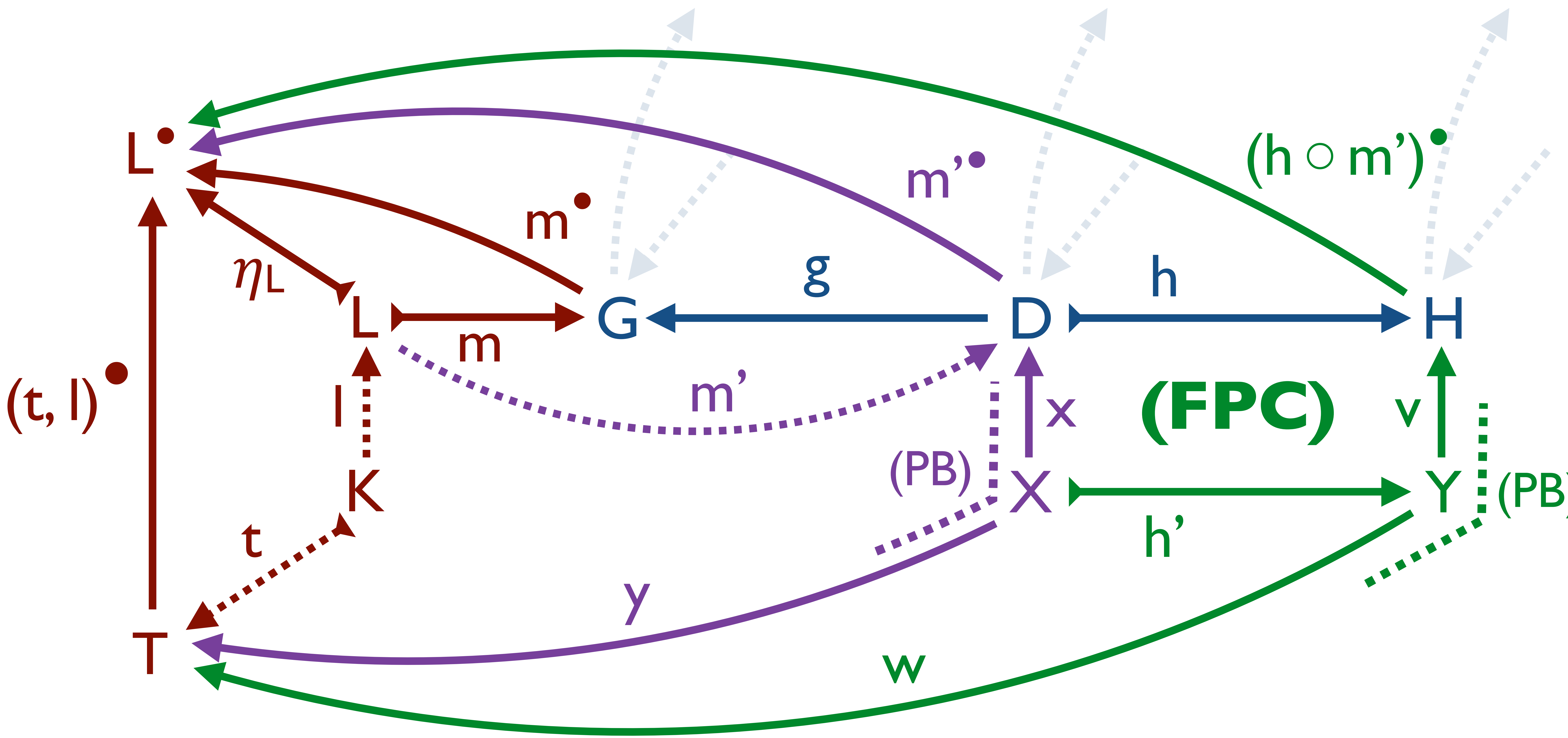


Residual



Residual



Characterising Independence

$$
\begin{tikzcd}
 & & L_1 \arrow[d, tail, "m_1"] & \arrow[l, dotted] \\
L_2^\bullet & L_2 \arrow[l, tail, "\eta_2"'] \arrow[r, tail, "m_2"'] & G & \\
T_2 \arrow[u, "(t_2, l_2)^\bullet"] \arrow[d, dotted] & K_2 \arrow[l, tail, "t_2"'] \arrow[u, "l_2"'] \arrow[d, dotted] & & \\
 & & &
\end{tikzcd}
$$

Characterising Independence

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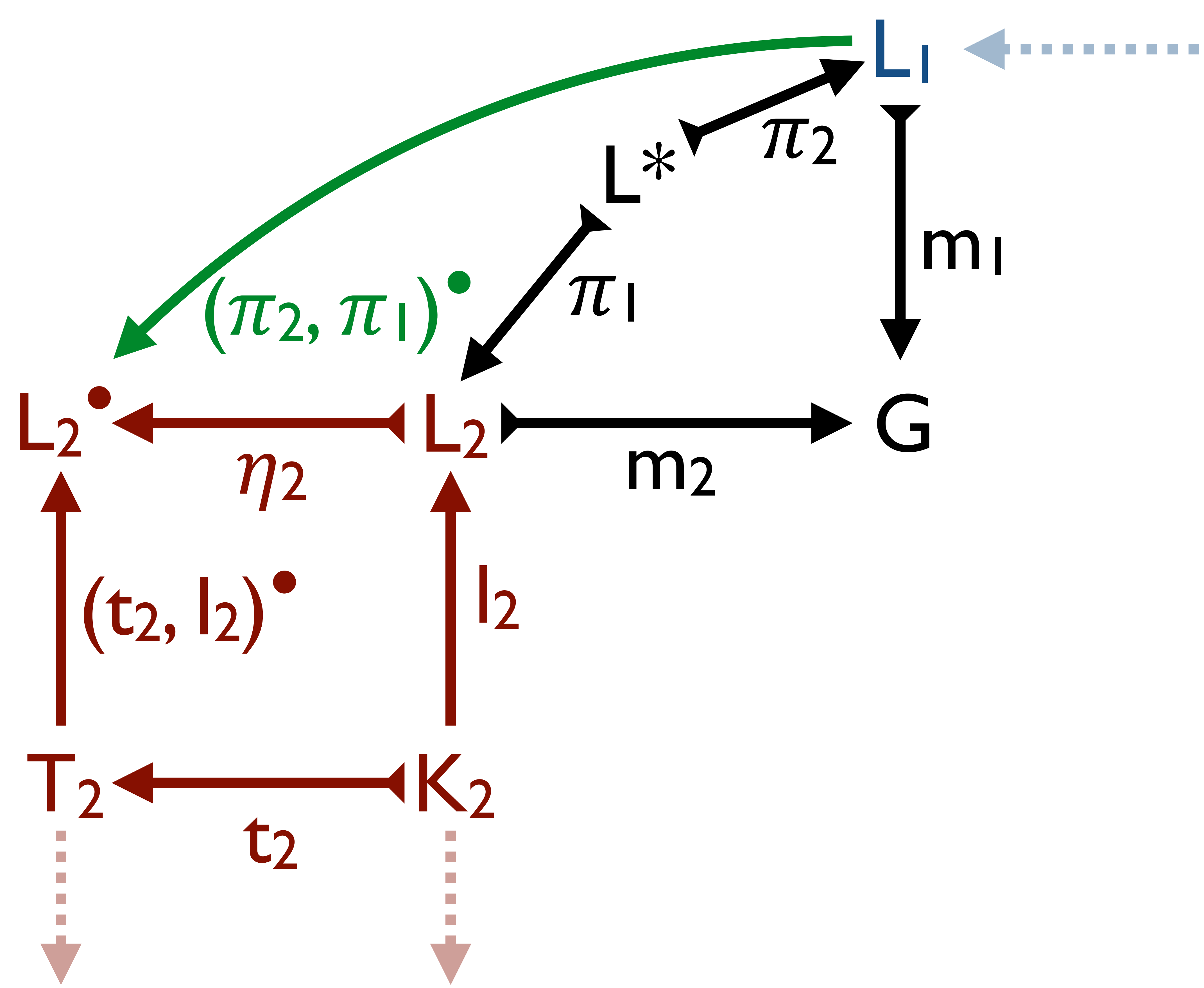
Match m_1 for rule 1 has residual after applying rule 2 at m_2, only if

1. everything that m_1 needs (locally copies, deletes, or preserves) is neither copied nor deleted (neither locally nor globally) by rule 2 at match m_2.

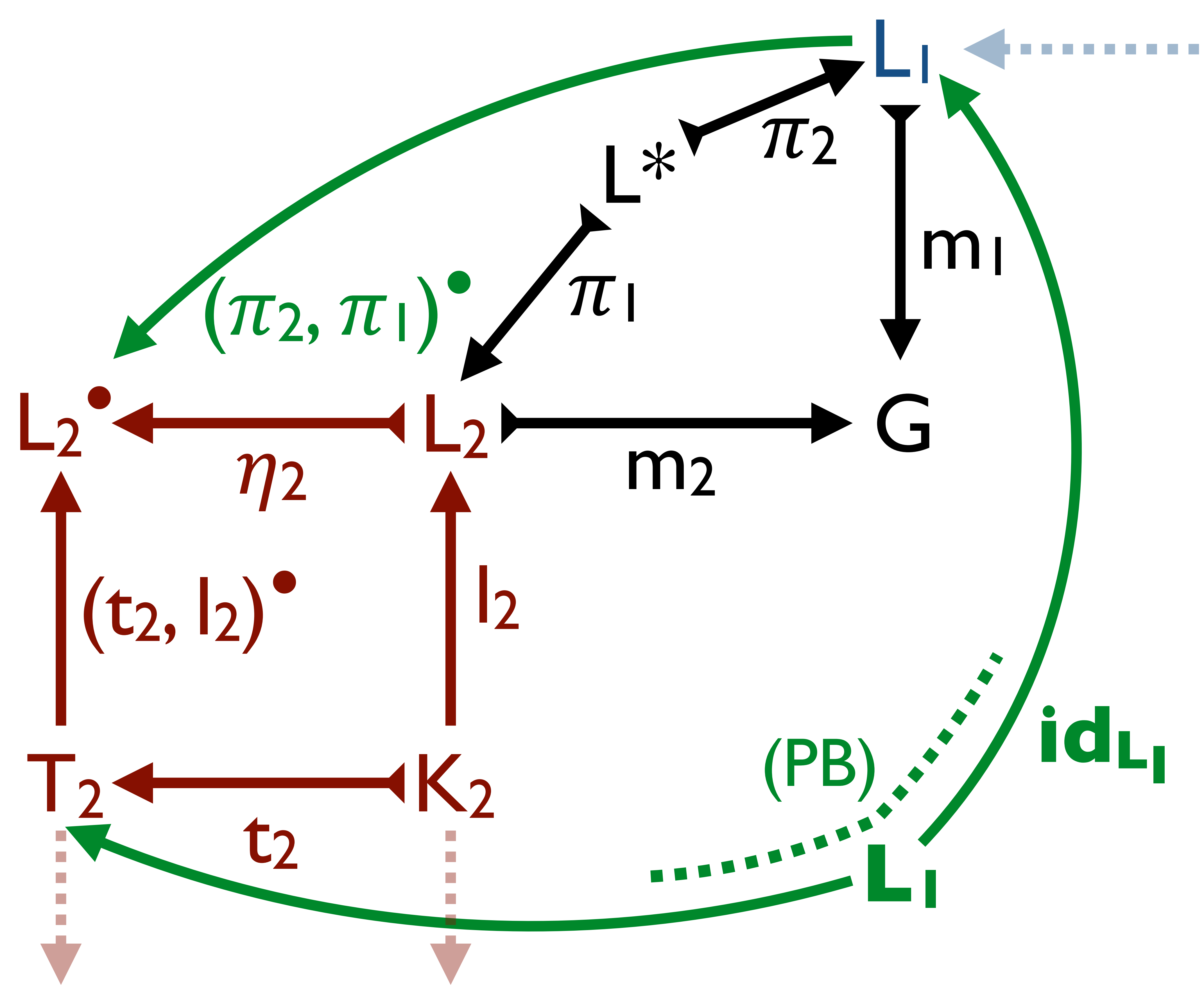
Characterising Independence

$$
\begin{tikzcd}
 & & L_1 \arrow[d, tail, "m_1"] & \arrow[l, dashed] \\
L_2^\bullet & L_2 \arrow[l, tail, "\eta_2"'] \arrow[r, tail, "m_2"'] & G & \\
T_2 \arrow[u, "(t_2, l_2)^\bullet"] \arrow[d, dashed] & K_2 \arrow[l, tail, "t_2"'] \arrow[u, "l_2"'] \arrow[d, dashed] & & \\
 & & &
\end{tikzcd}
$$

Characterising Independence



Characterising Independence



Characterising Independence

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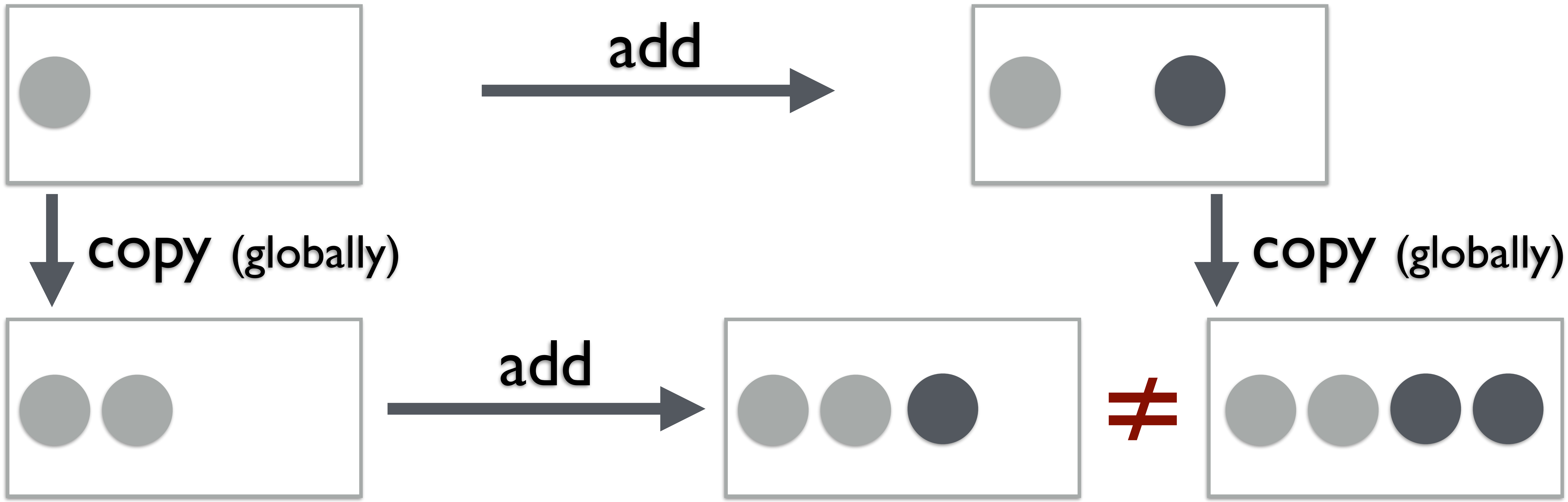
Match m_1 for rule 1 has residual after applying rule 2 at m_2, only if

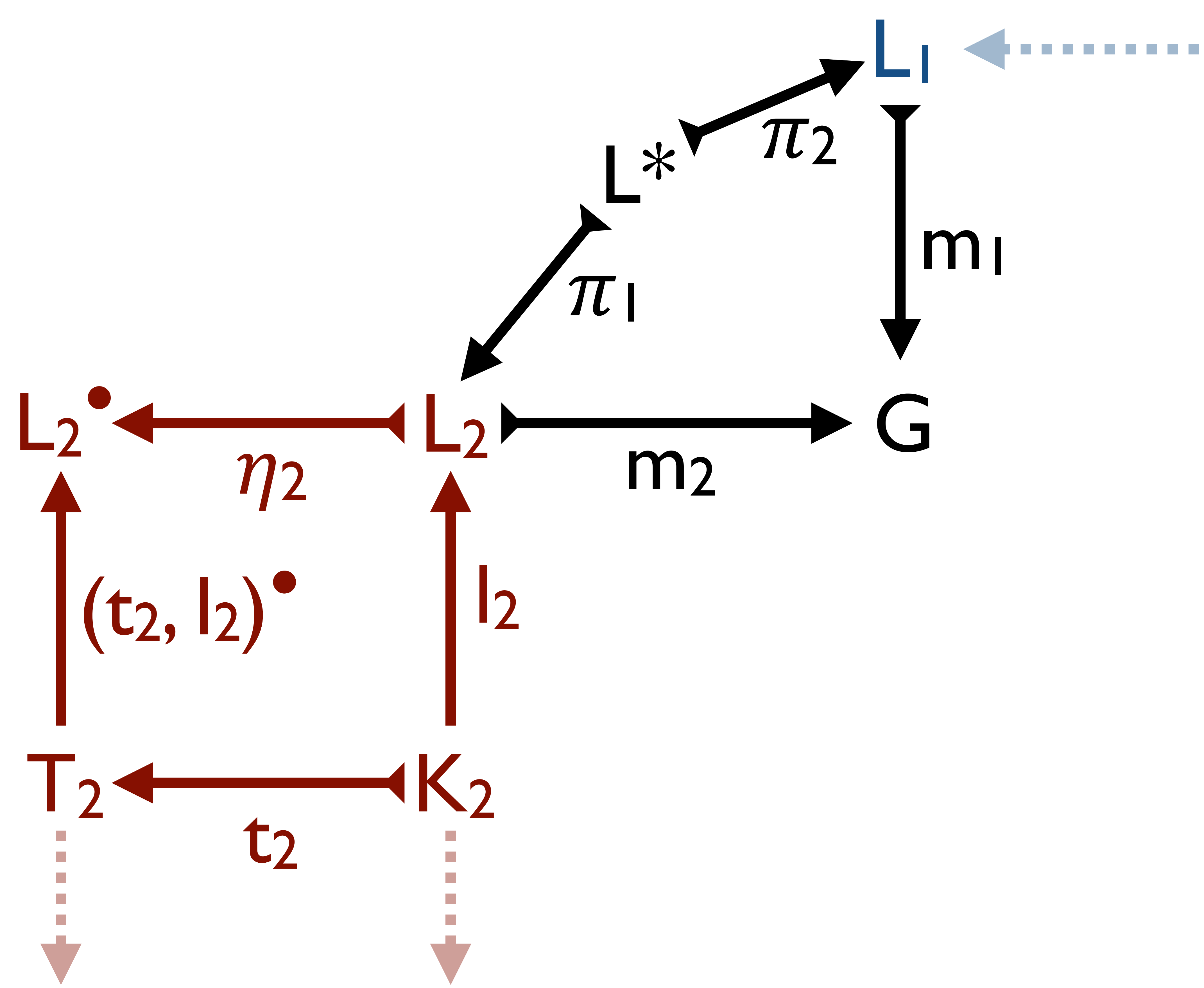
1. everything that m_1 needs (locally copies, deletes, or preserves) is neither copied nor deleted (neither locally nor globally) by rule 2 at match m_2.
2. everything that rule 1 adds is neither (globally) copied nor deleted by rule 2 at match m_2.

Characterising Independence

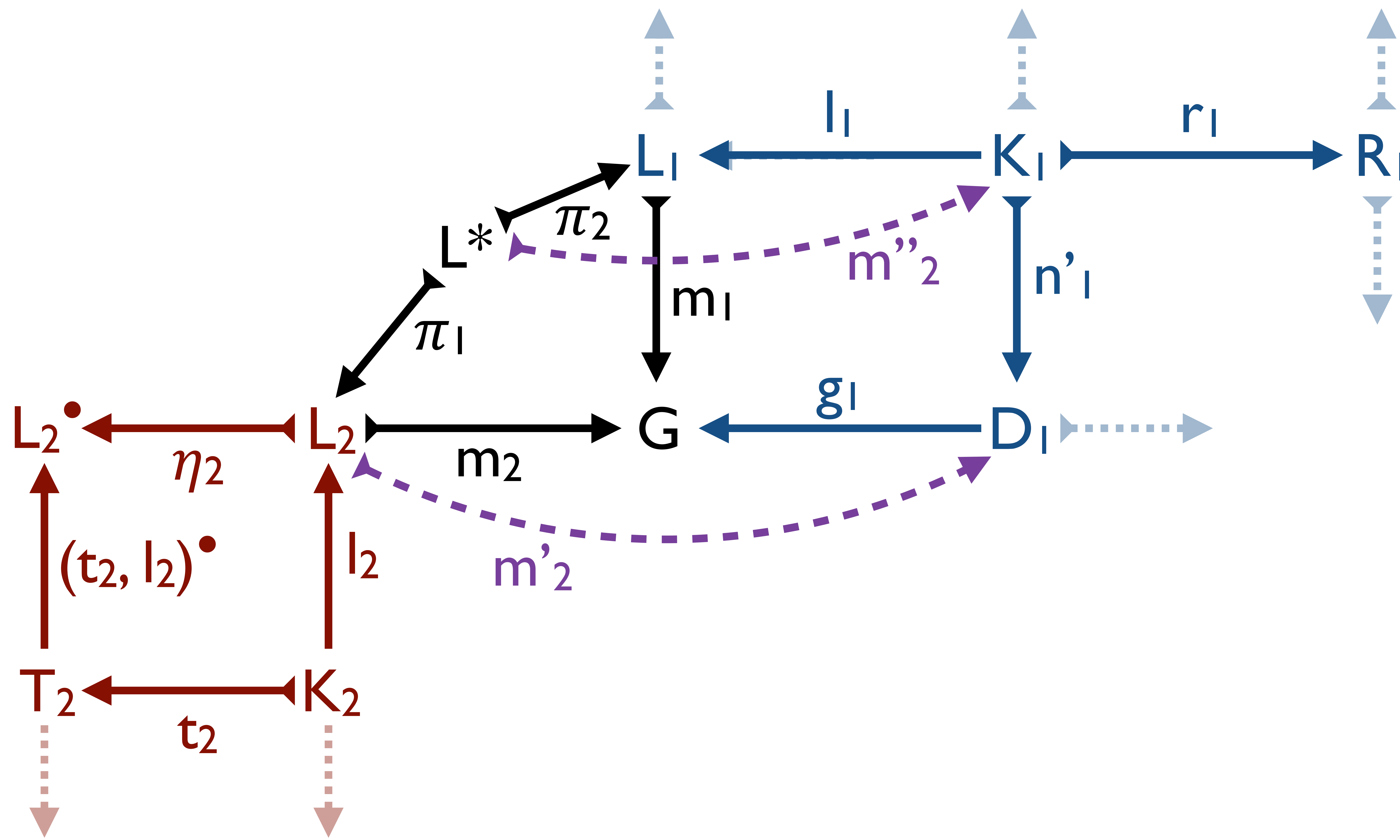


2. everything that rule 1 adds is neither (globally) copied nor deleted by rule 2 at match m_2.

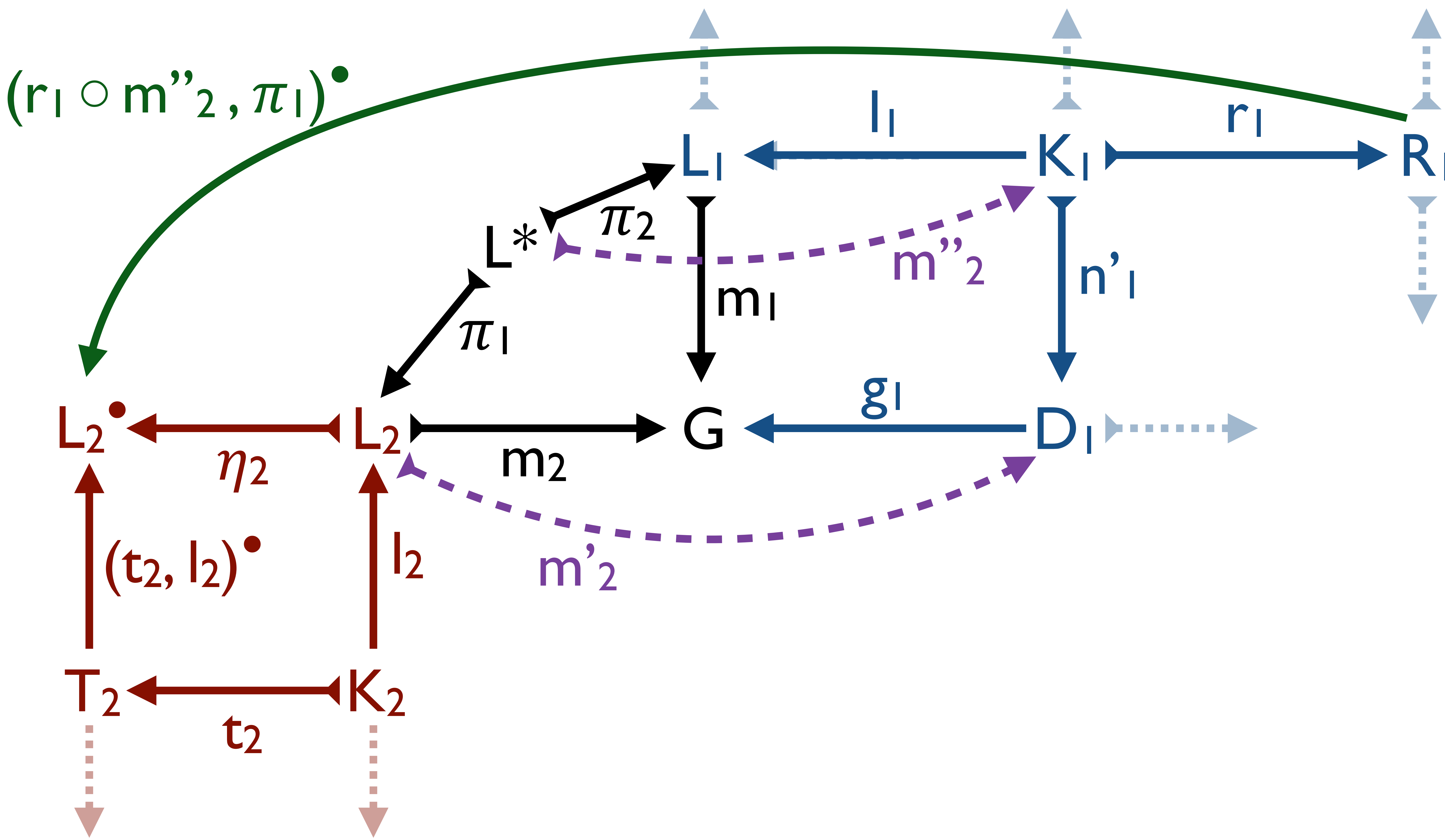
Characterising Independence



Characterising Independence



Characterising Independence



Characterising Independence

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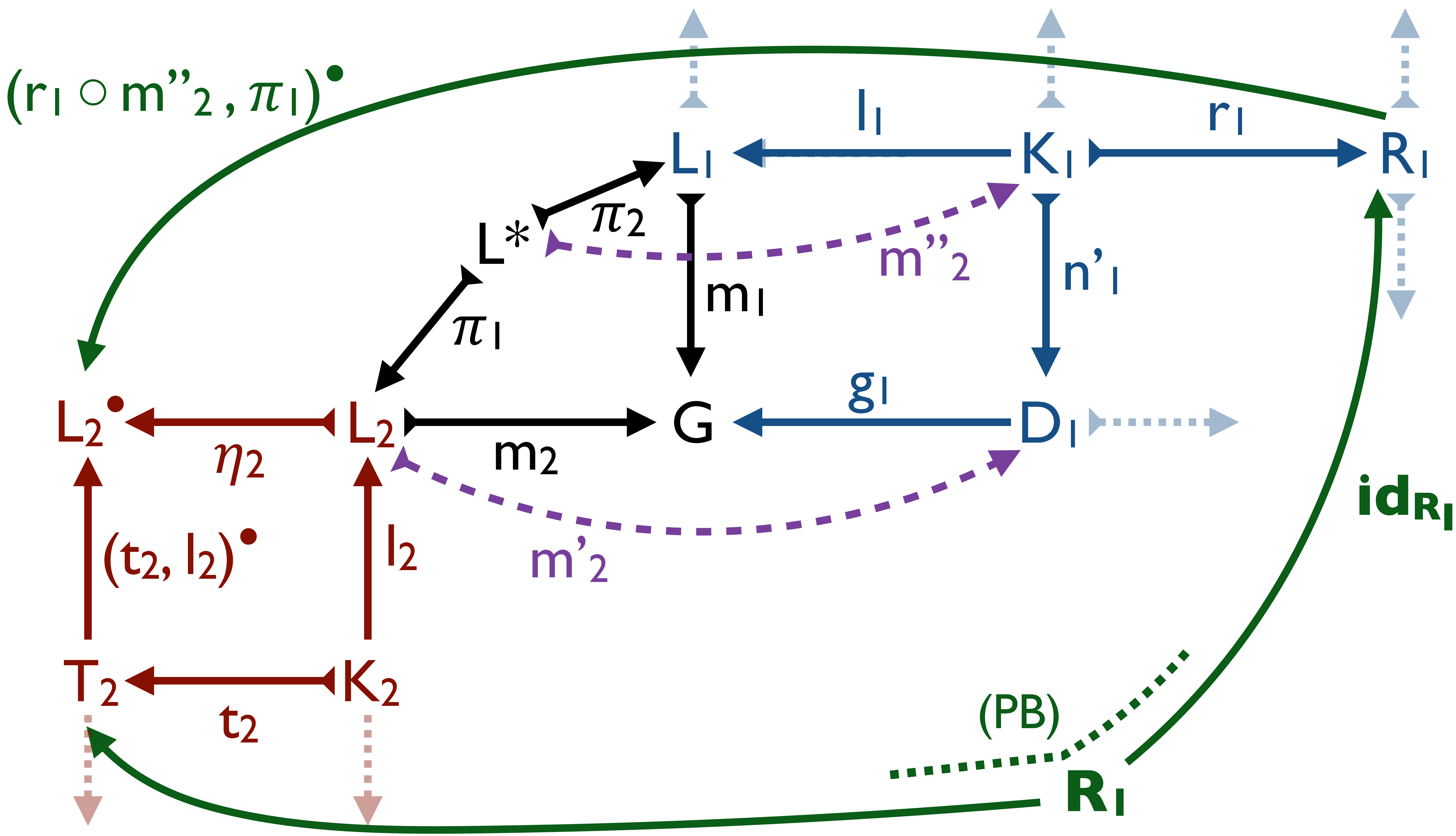

Characterising Independence

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HANNOVER

Match m_1 for rule 1 has residual after applying rule 2 at m_2, **if and only if**

1. everything that m_1 needs (locally copies, deletes, or preserves) is neither copied nor deleted (neither locally nor globally) by rule 2 at match m_2.
2. everything that rule 1 adds is neither (globally) copied nor deleted by rule 2 at match m_2.

Characterising Independence

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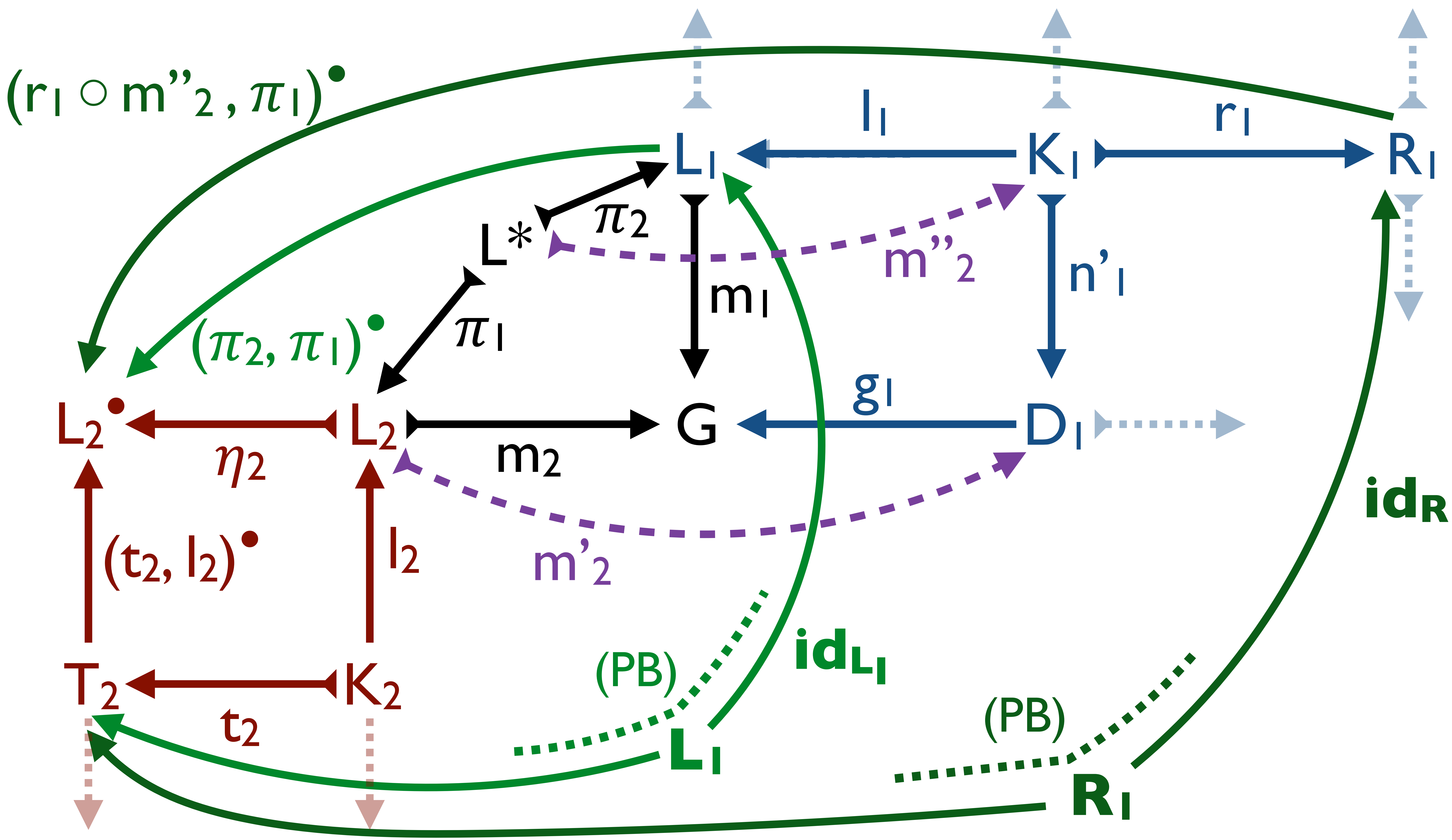

Conclusion

AGREE-rewriting is instance of the Gluing Construction!

There is a precise notion of residual!

Gluing and mutual residuals provides Church-Rosser!

Residuals can be characterized syntactically!

Are global effects useful?

Thank you for your attention