On the Essence and Initiality of Conflicts

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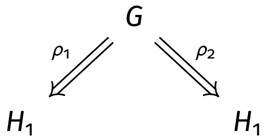
11th International Conference on Graph Transformation,
June 2018

Parallel Independence
of Transformations



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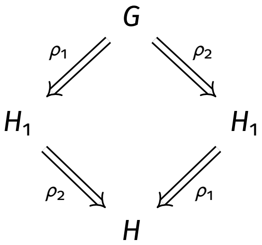

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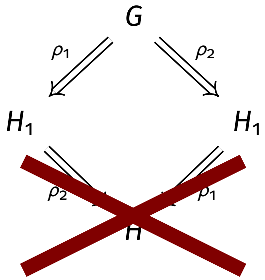


Conflict

Motivation

- Conflicts capture important information about behaviour
- Enumerating **potential conflicts** has many applications
  - Critical pairs or initial conflicts
- Understanding **root causes** is often important

Background: The DPO Approach

Rule: $\rho = L \overset{l}{\leftarrowtail} K \overset{r}{\rightarrowtail} R$

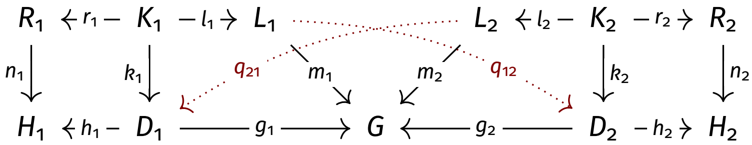
Match: $m : L \rightarrowtail G$

Transformation: $G \overset{\rho,m}{\Longrightarrow} H$

$$\begin{array}{ccccc} L & \xleftarrow{l} & K & \xrightarrow{r} & R \\ \downarrow m & \text{PO} & \downarrow k & \text{PO} & \downarrow n \\ G & \xleftarrow[g]{} & D & \xrightarrow[h]{} & H \end{array}$$

New Perspective

- Previous work based on the **standard condition** for parallel independence



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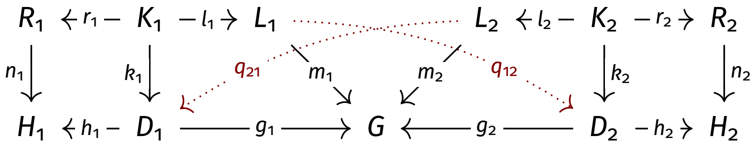

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- Equivalent to standard condition

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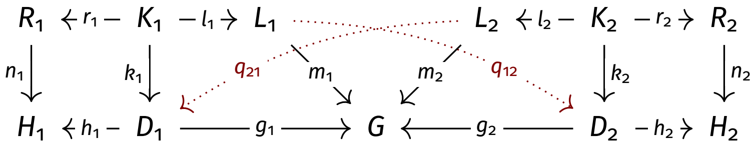
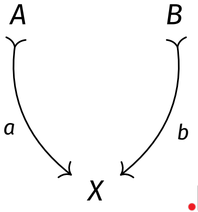


- Recently: **essential condition** for parallel independence (Corradini et al. 2018)
- Equivalent to standard condition
- **Goal:** review characterization of conflicts under new light

Background: Adhesive Categories

Subobjects behave like subsets



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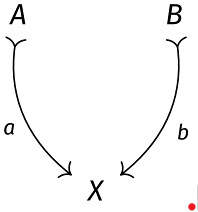
Subobjects behave like subsets

Lemma (Lack and Sobocinski 2005)

In adhesive categories, $\mathbf{Sub}(X)$ *is distributive lattice*



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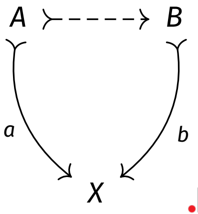

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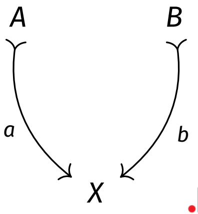

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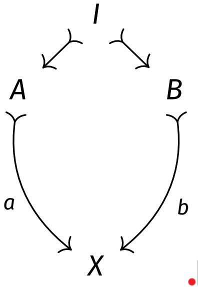

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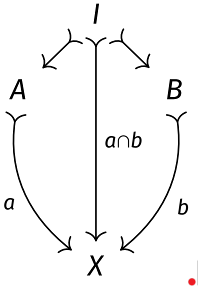

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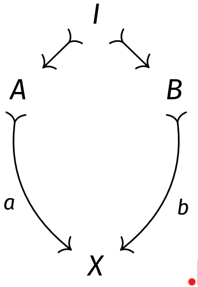

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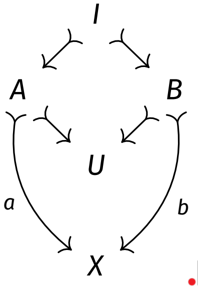

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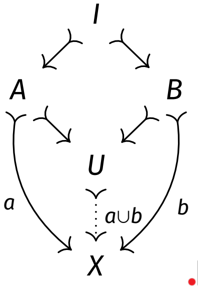

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Intersection	pullback
Union	pushout over intersection
Top	is X
Bottom	usually "empty", if exists

Background: Set-Valued Functor Categories

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- We use categories $\mathbb{S}\text{et}^{\mathbb{S}}$ of functors $\mathbb{S} \to \mathbb{S}\text{et}$ with natural transformations as arrows (essentially presheaves)
- Generalizes graphs and graph structures

$$\mathbb{G}\text{raph} = \mathbb{S}\text{et}^{\mathbb{G}} \qquad \mathbb{G} = \quad V \overset{s}{\underset{t}{\rightrightarrows}} E$$

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$$\mathbb{G}\text{raph} = \mathbb{S}\text{et}^{\mathbb{G}} \qquad \mathbb{G} = V \underset{t}{\overset{s}{\rightrightarrows}} E$$

- Limits, colimits, monos and epis are pointwise
- Always adhesive

Outline

1. Characterize conflict between transformations
2. Useful properties of the characterization
3. Compare with conflict reasons of Lambers, Ehrig, and Orejas (2008)
4. Relate to initial conflicts

Essential Condition of Parallel Independence

Corradini et al. (2018)

$$H_1 \overset{t_1}{\Longleftarrow} G \overset{t_2}{\Longrightarrow} H_2$$

$$\begin{array}{ccccccccccccc}
R_1 & \xleftarrow{r_1} & K_1 & \xrightarrow{l_1} & L_1 & & & & L_2 & \xleftarrow{l_2} & K_2 & \xrightarrow{r_2} & R_2 \\
{\scriptstyle n_1}\downarrow & & {\scriptstyle k_1}\downarrow & & & \searrow^{m_1} & & {}^{m_2}\swarrow & & & \downarrow{\scriptstyle k_2} & & \downarrow{\scriptstyle n_2} \\
H_1 & \xleftarrow{h_1} & D_1 & & \xrightarrow{g_1} & & G & & \xleftarrow{g_2} & & D_2 & \xrightarrow{h_2} & H_2
\end{array}$$

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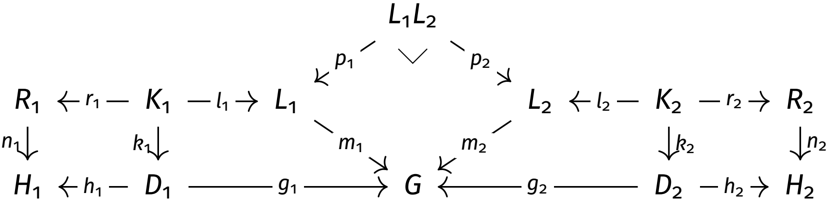

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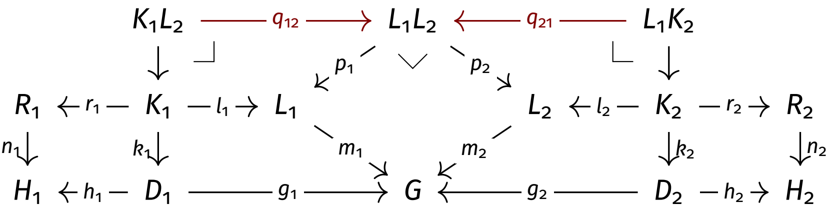

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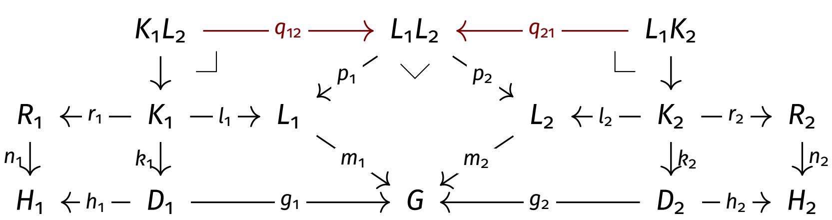

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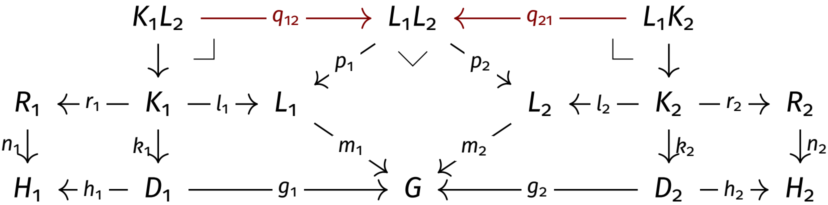

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- Either morphism not iso ⇒ conflict

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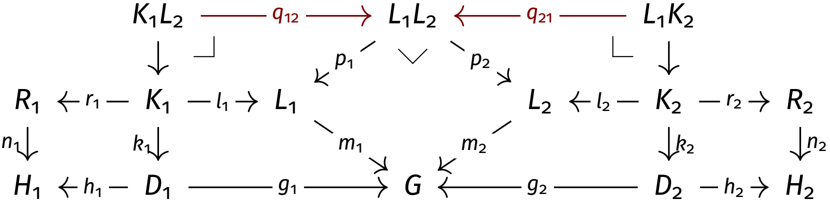
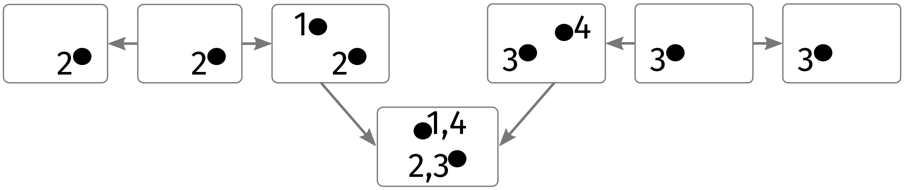


- Both morphisms iso $\Rightarrow$ parallel independence
- Either morphism not iso $\Rightarrow$ conflict
- $K_1L_2 \to L_1L_2$ not iso $\Rightarrow$ t_1 disables t_2

Example: Conflict



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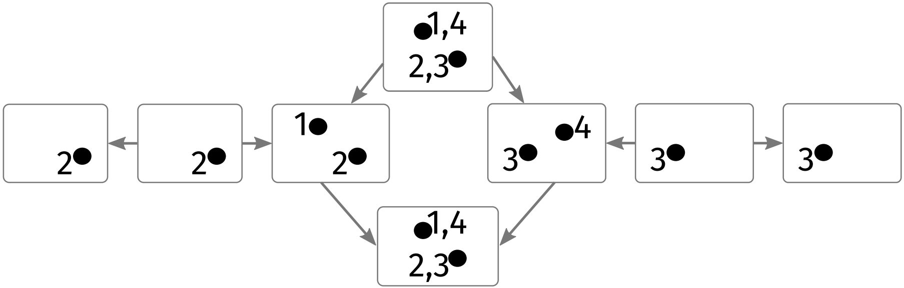

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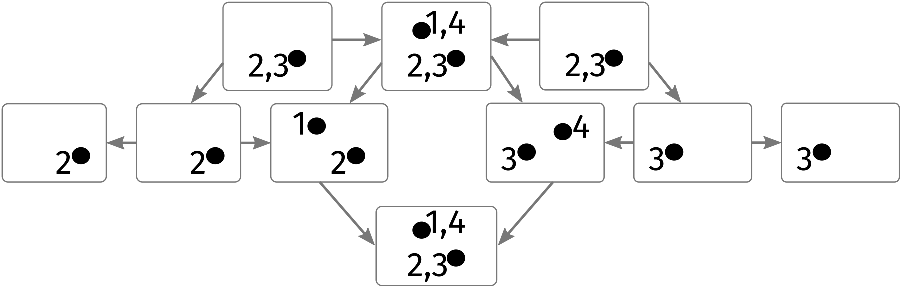

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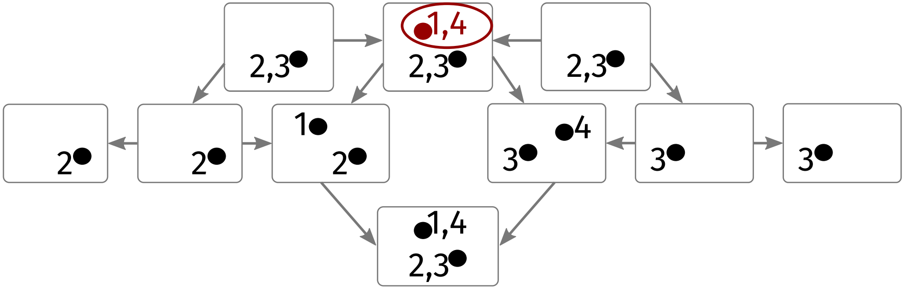

Determining the Root Cause

- Useful concept: initial pushout over $f : X \to Y$

$$\begin{array}{ccc} B & \overset{b}{\rightarrowtail} & X \\ {\scriptstyle \bar{f}}\downarrow & & \downarrow{\scriptstyle f} \\ C & \underset{c}{\rightarrowtail} & Y \end{array}$$

- “Categorical diff” for a morphism

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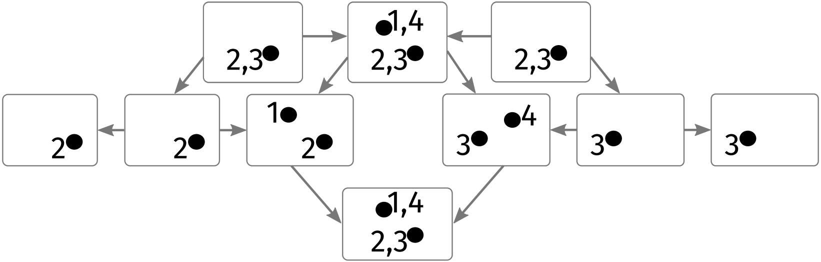
- "Categorical diff" for a morphism
- Context $c : C \rightarrowtail Y$ contains "modified stuff"
- Boundary $b : B \rightarrowtail C$ contains "points of contact"

Example: Initial Pushout



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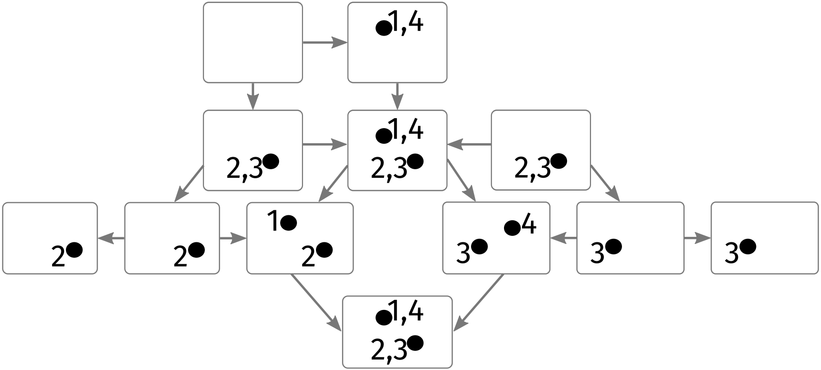

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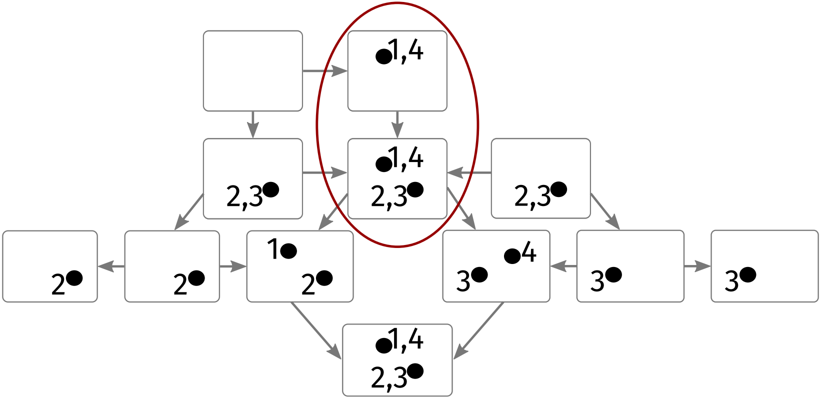
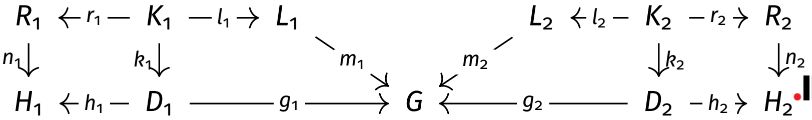


Conflict and Disabling Essences

Definition

Given transformations $(t_1, t_2) : H_1 \overset{\rho_1, m_1}{\Longleftarrow} G \overset{\rho_2, m_2}{\Longrightarrow} H_2$:



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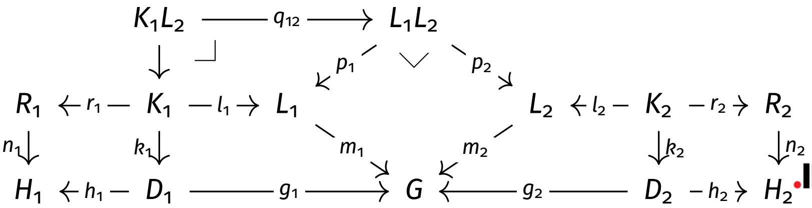

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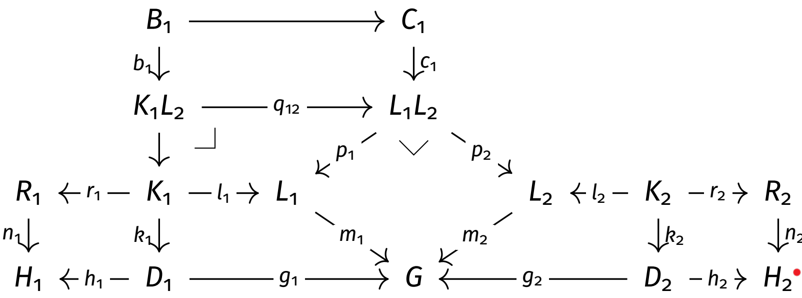

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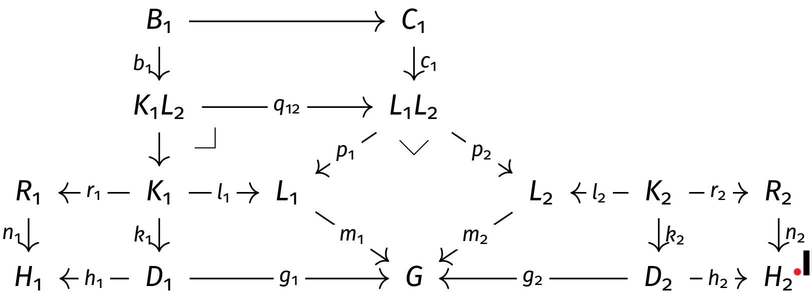

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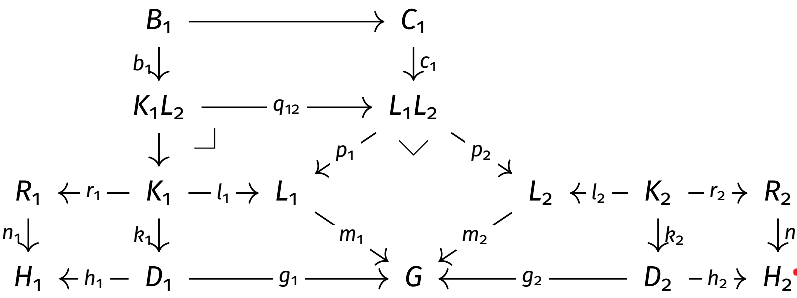

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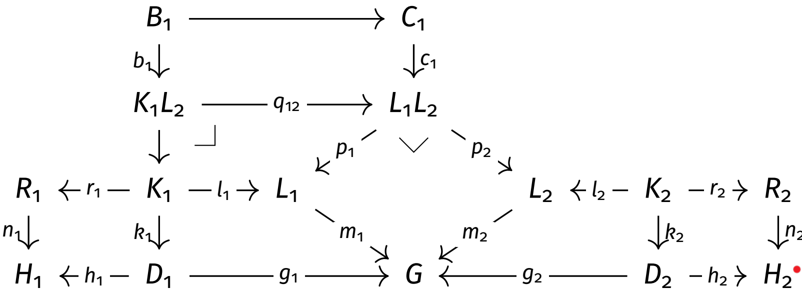
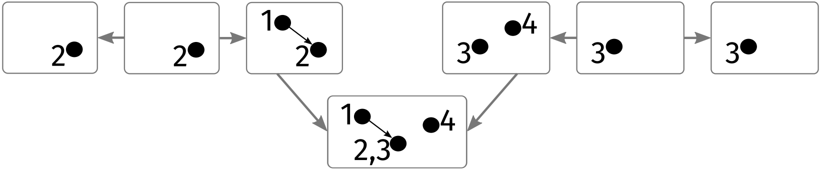
- Disabling essence for (t_2, t_1) is $c_2 \in \textbf{Sub}(L_1 L_2)$
- Conflict essence for (t_1, t_2) is $c = c_1 \cup c_2$



Example: Parallel Independence



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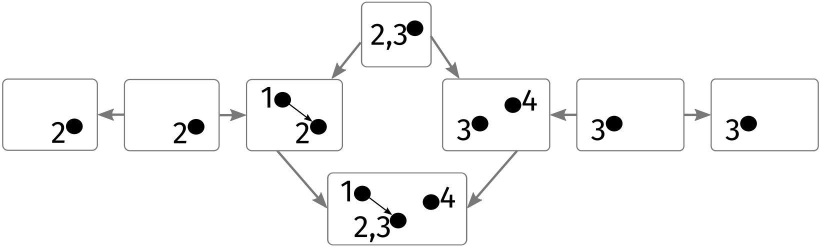

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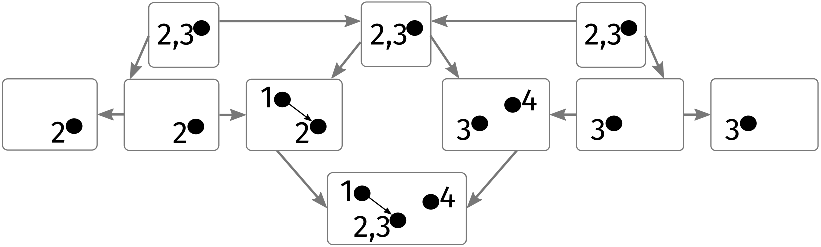

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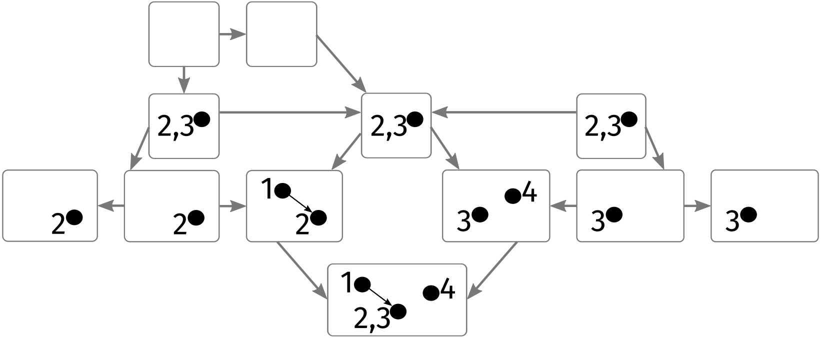

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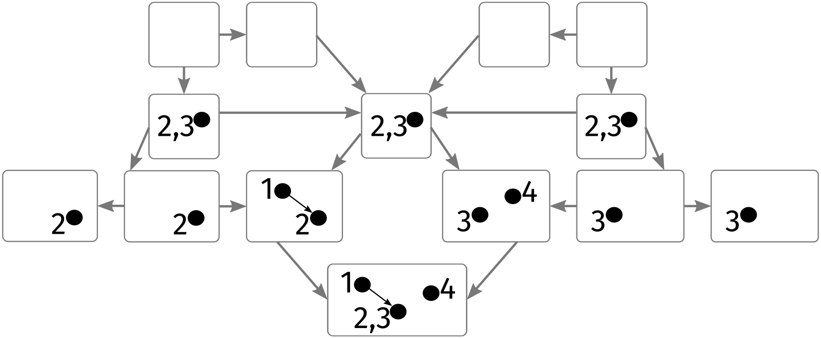

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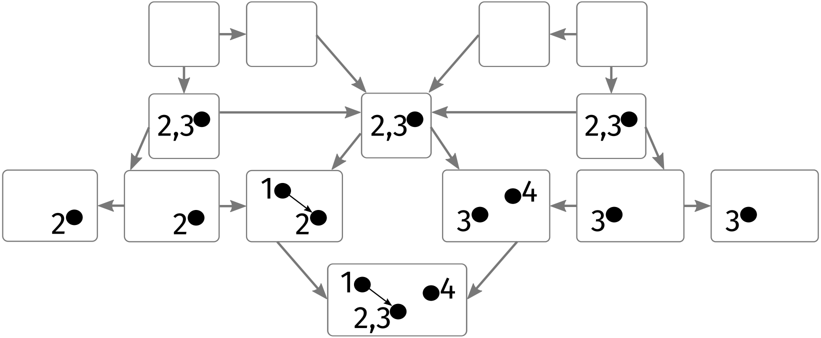

No conflict $\Longrightarrow$ no element caused a conflict

Empty Essences

Recall: bottom subobject generalizes “emptiness”

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$$\text{Consider } (t_1, t_2) : H_1 \overset{\rho_1, m_1}{\Longleftarrow} G \overset{\rho_2, m_2}{\Longrightarrow} H_2$$

Theorem

The conflict essence for (t_1, t_2) is $\bot \in \mathbf{Sub}(L_1 L_2)$
if and only if
t_1 and t_2 are parallel independent.

Extension

- Same transformation in "larger context"

$$
\begin{array}{ccc}
\overline{G} & \overset{\overline{t}}{\Longrightarrow} & \overline{H} \\
\downarrow f & & \downarrow f' \\
G & \overset{t}{\Longrightarrow} & H
\end{array}
\equiv
\begin{array}{ccccc}
L & \xleftarrow{l} & K & \xrightarrow{r} & R \\
\downarrow \overline{m} & & \downarrow \overline{k} & & \downarrow \overline{n} \\
\overline{G} & \xleftarrow{\overline{g}} & \overline{D} & \xrightarrow{\overline{h}} & \overline{H} \\
\downarrow f & & \downarrow d & & \downarrow f' \\
G & \xleftarrow{g} & D & \xrightarrow{h} & H
\end{array}
$$

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\downarrow f & & \downarrow d & & \downarrow f' \\
G & \xleftarrow{g} & D & \xrightarrow{h} & H
\end{array}
$$

- Lower pushouts ensure t behaves like $\bar{t}$

Essence Inheritance

Theorem

If extension diagrams below exist, (t_1, t_2) and $(\overline{t_1}, \overline{t_2})$ have the same disabling and conflict essences.

$$
\begin{array}{ccccc}
\overline{H_1} & \overset{\overline{t_1}}{\Longleftarrow} & \overline{G} & \overset{\overline{t_2}}{\Longrightarrow} & \overline{H_2} \\
\downarrow & & \downarrow f & & \downarrow \\
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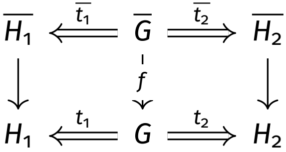

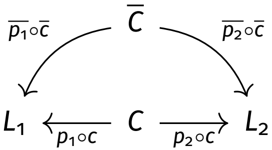

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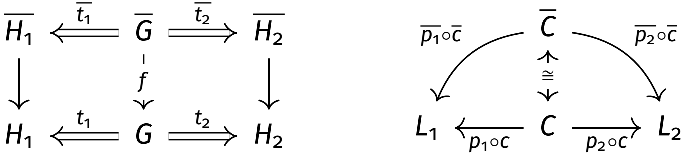

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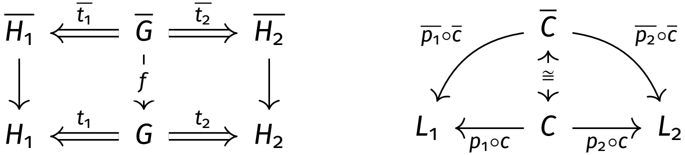

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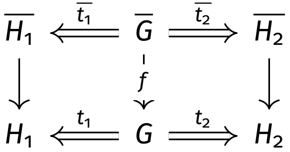

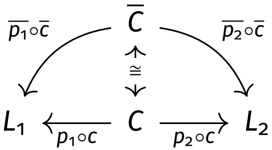

Conflicts are preserved and reflected by extension.

Disabling Reasons

Essences are not the first proposed characterization

Given transformations $(t_1, t_2) : H_1 \overset{\rho_1, m_1}{\Longleftarrow} G \overset{\rho_2, m_2}{\Longrightarrow} H_2$

Definition (Lambers, Ehrig, and Orejas 2008)

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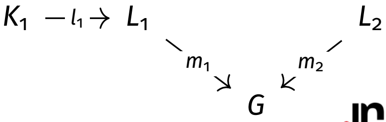

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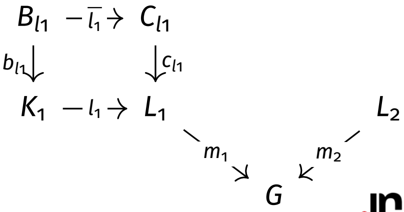

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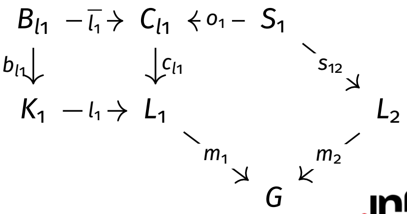

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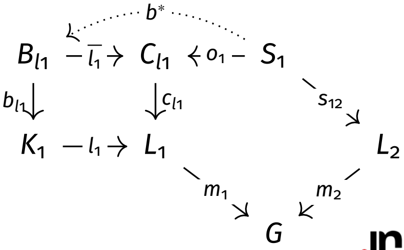

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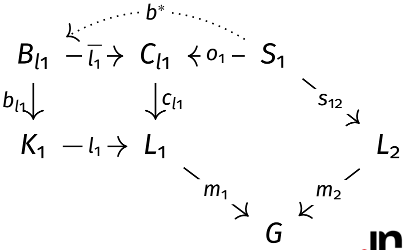
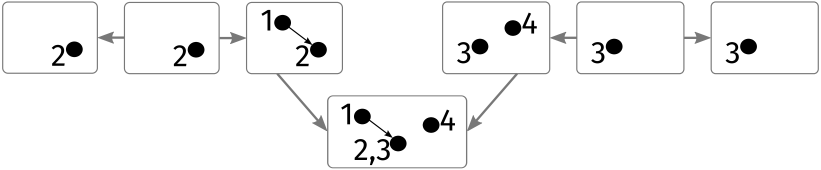
Conflict reason is union of *relevant* disabling reasons.



Comparing Reasons and Essences

- Non-empty reasons exist even with parallel independence



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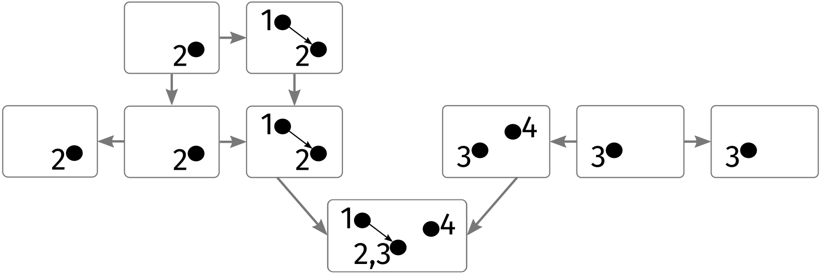

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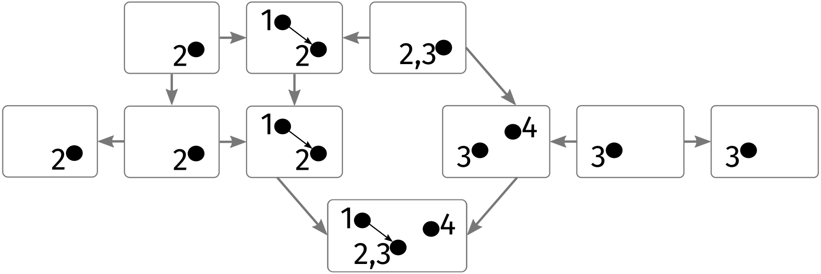

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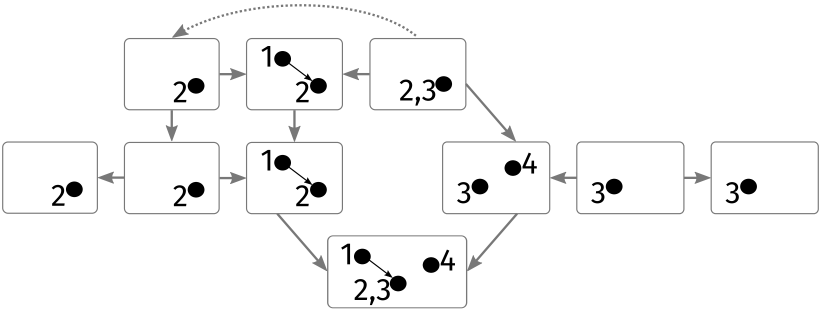

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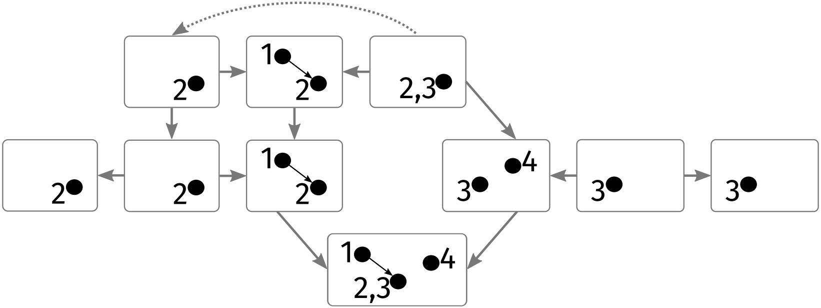

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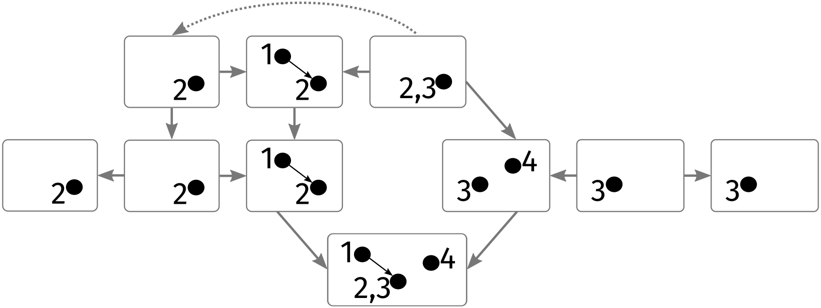

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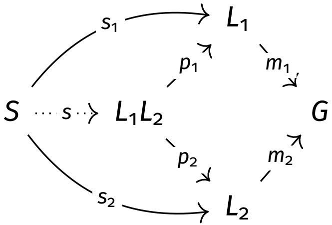
Conflict reason determines $s \in \mathbf{Sub}(L_1 L_2)$.



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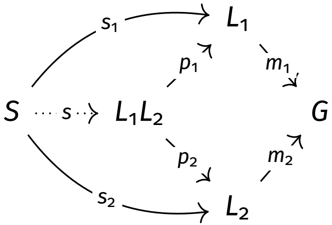

Theorem

If $c \in \mathbf{Sub}(L_1 L_2)$ is disabling essence and $s \in \mathbf{Sub}(L_1 L_2)$ disabling reason, then $c \subseteq s$.
The same holds if c is conflict essence and s conflict reason.

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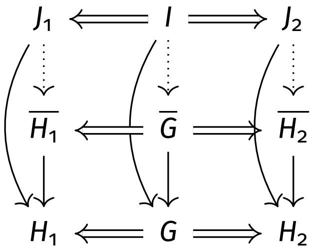

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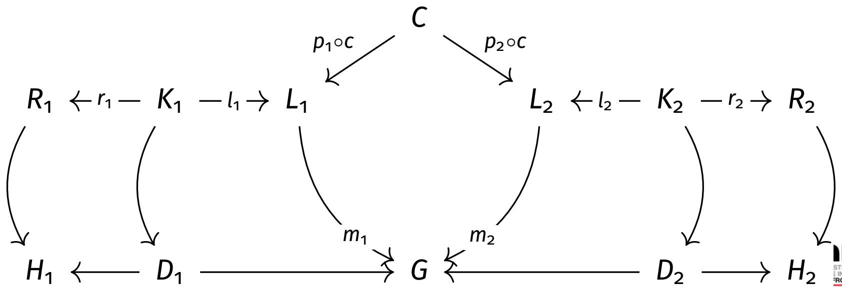

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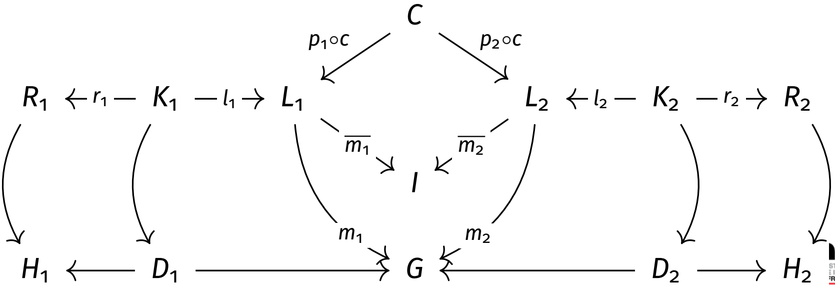

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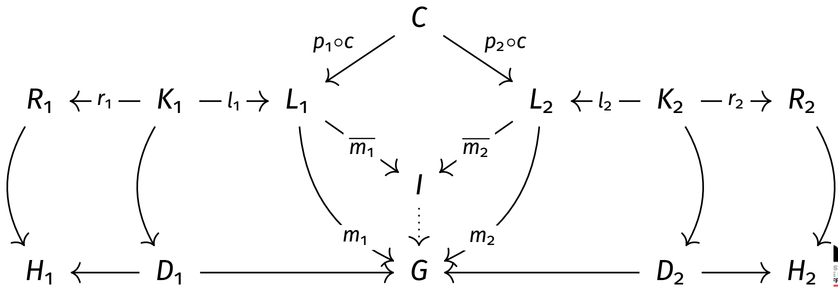

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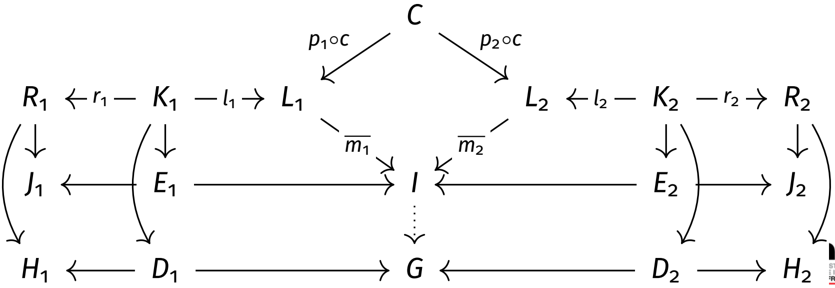

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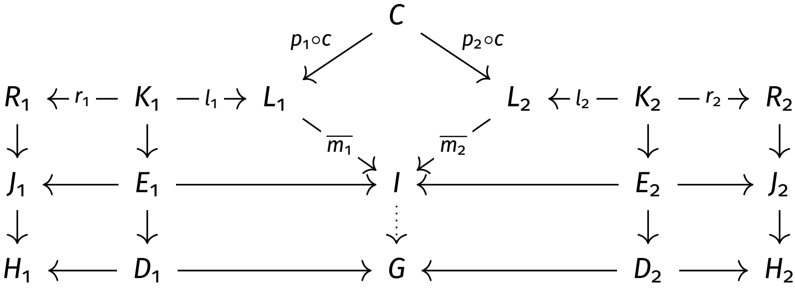

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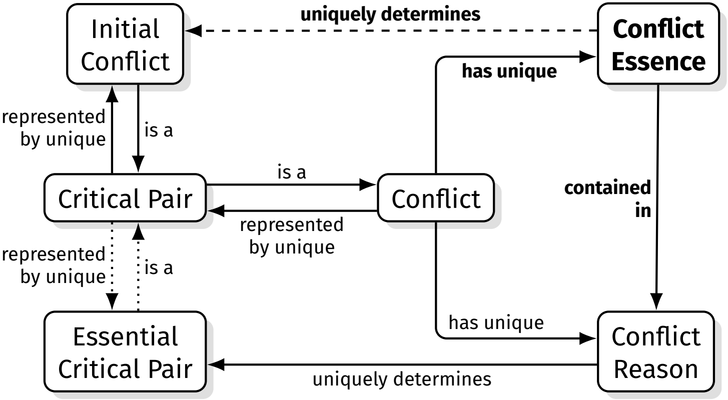

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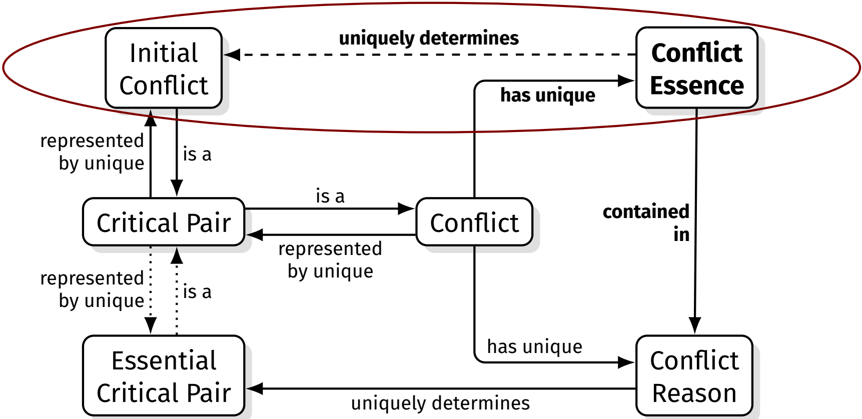

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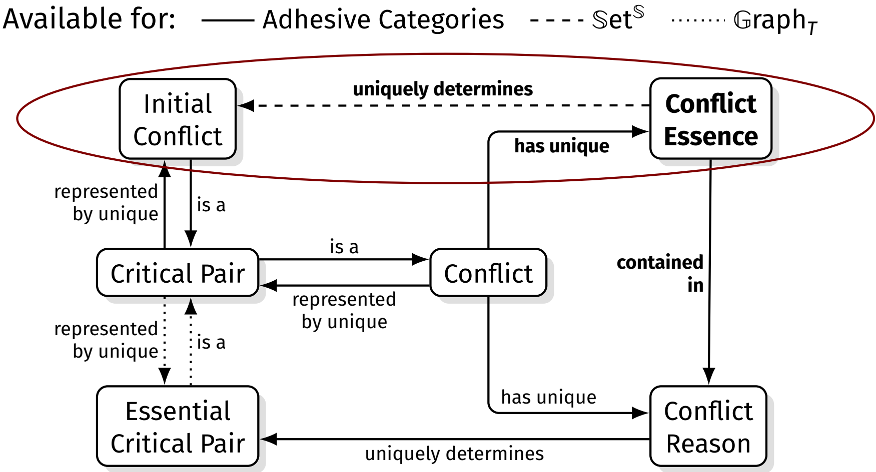

Open Problem: in all adhesive categories?

Conclusions

- Essential condition allowed powerful characterization for root causes of conflicts
- Lots of future work!
  - Constraints and application conditions
  - Compare with notions of granularity (Born et al. 2017)
  - Attributed graphs and other adhesive categories
  - Sesqui-Pushout and AGREE

Thank you!

Questions?

References I

Born, Kristopher et al. (2017). "Granularity of Conflicts and Dependencies in Graph Transformation Systems". In: *ICGT*. Vol. 10373. LNCS. Springer, pp. 125–141. DOI: 10.1007/978-3-319-61470-0_8. URL: https://doi.org/10.1007/978-3-319-61470-0_8.

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Notes